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Height Reference Surface Modelling and Computation

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Summary

The satellite-based measuring technologies are getting bigger and bigger role in daily surveying tasks. Whether we speak about on-line or post-processing, the determination of the height is always a key question: 'how can we derive precise standard heights from ellipsoidal heights provided by satellite measurements?' The mathematical relationship between these two quantities is very simple, but not the way of the realisation. Hundreds of studies have been published in the past two decades in this field of the Geodesy but leaving behind always open questions. These open questions refer to the correctness and the approximations of the mathematical formulisation, and last but not least, to the applicability.

Most of the studies just discuss the GNSS/levelling data like only a control to check the goodness of the gravimetric solutions omitting the most precise information in the geoid computations, namely the heights. Combined GNSS and levelling data can be utilised to replace the laborious levelling measuring technology over a certain distance. This has been aimed in many countries. However, this method requires a precise GNSS/levelling frame that can be built on a zero or first order levelling network. These networks must be maintained in the future as well to keep the relation between the standard and GNSS ellipsoidal heights. Consequently, the precision of GNSS levelling highly depends on the reliability of a present levelling network, and because of the tectonic motions, so will it in the future.

The Digital Finite Element Height Reference Surface (DFHRS) Method

The Functional Model

Standard height $H + v_H = \hat{H}$

Ellipsoidal height $\hat{h} = \hat{H} + f(\hat{\mathbf{p}})$

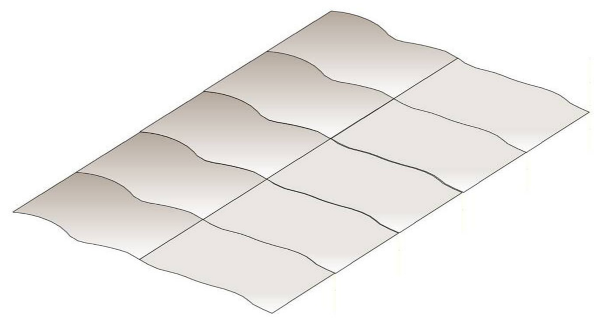
Geoid model $\hat{N}_{GM} = f(\hat{\mathbf{p}}) + dN_{GM}(\hat{\mathbf{d}})$

Astronomical Observations

$$\hat{\xi} = -\frac{\mathbf{f}_\varphi^T}{R_M}\cdot\hat{\mathbf{p}} + d\xi(\hat{\mathbf{d}}) \qquad \hat{\eta} = -\frac{\mathbf{f}_\lambda^T}{R_N\cos\varphi}\cdot\hat{\mathbf{p}} + d\eta(\hat{\mathbf{d}})$$

For on-line GNSS heighting and height reference surface database concept, the best mathematical formulisation is that contains all available data playing key role in the determination of height reference surface. These are

- standard heights
- ellipsoidal heights
- global / regional geoid models
- astronomical observations
- gravity data



Gravity data

$$\Delta g + v_{\Delta g} = \frac{GM}{a^2}\sum_{n=2}^{\infty}(n-1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left(\delta C_{nm}\cos m\lambda' + \delta S_{nm}\sin m\lambda'\right)\cdot P_{nm}(\cos s)$$

$$0 + v_{\Delta N} = \frac{GM}{a\gamma}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left(\delta C_{nm}\cos m\lambda' + \delta S_{nm}\sin m\lambda'\right)\cdot P_{nm}(\cos s) - f(\hat{\mathbf{p}})$$

Three Dimensional Finite Element Height Reference Surface Modelling Based On Geoid Refinement Approach

The geoid refinement approach tends to reduce the existing systematic errors being in global or regional geoid model. The systematic errors may be classified according to their entering point into the entire geoid computation process. They can be measuring errors and model errors as well. For example, during the course of orthometric height computations, we must take assumptions on the density distribution inside the earth. Systematic errors appear in the ellipsoidal heights, gravity data and astronomical observations too. These systematic errors cannot be modelled individually therefore we drop them in the bucket modelling them together. That means, we must correct the gravimetrically derived regional or global geoid. This leads to the introduction of the so-called corrector surface, or in other words, to the concept of the geoid refinement approach.

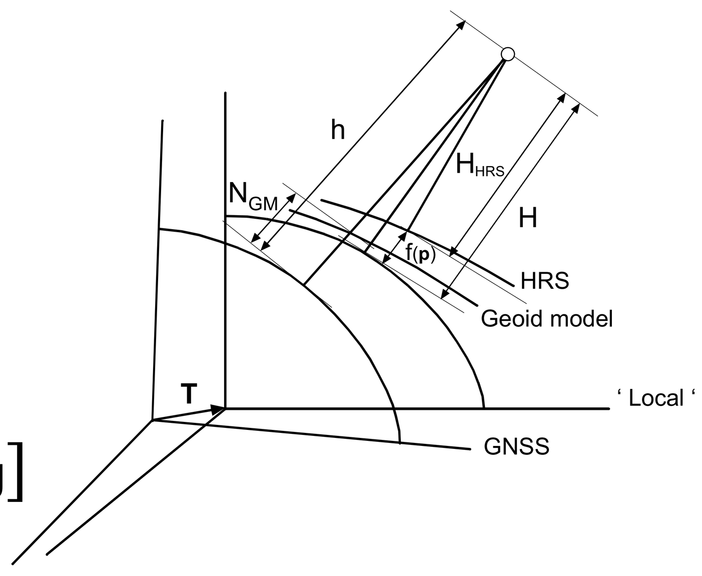
The Functional Model

Corrector surface

$N_{GM} + f(\mathbf{p}) = h - H$

Control points

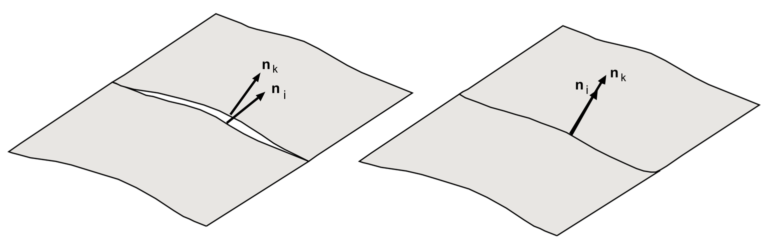
$\mathbf{X}_H = \mathbf{T} + \mathbf{R}\cdot\left[\mathbf{X}_{ho} + \left(h - N_{GM} - f(\mathbf{p})\right)\cdot\mathbf{g}\right]$



Continuity conditions

$v = f_i(\mathbf{p}) - f_k(\mathbf{p})$

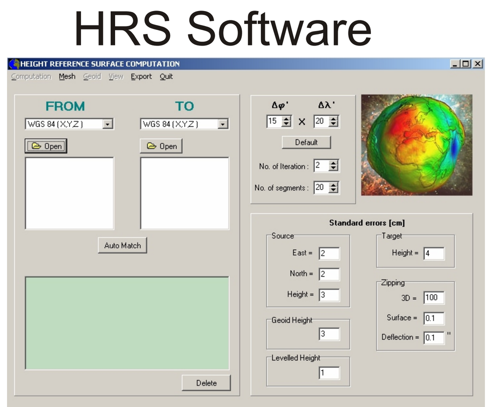
$\mathbf{n}_1 \times \mathbf{n}_2 = 0$



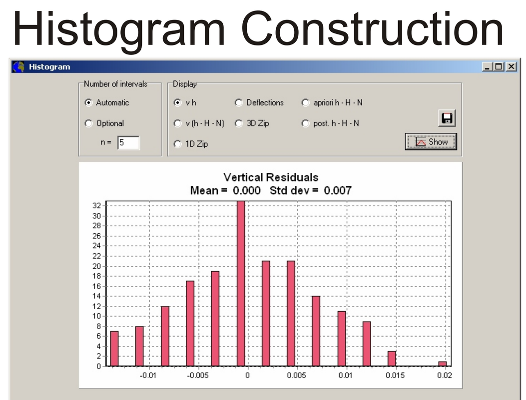
Application Development

The theory of three dimensional height reference surface method requires complex computations. The mathematical formulisation of the problem is only one thing that does not have any worth without practical realisation. Therefore I developed own software to solve this task effectively. The heart of the adjustment comprises 5 non-standard functions and 36 procedures. Dozens of furthermore functions and procedures were made to support the graphical visualisation as well. Histograms and plots have been developed in order to solve the representation of the results quickly and exporting them into windows metafile or bitmap format. I named my software simply HRS referring to the task that may be solved by it.

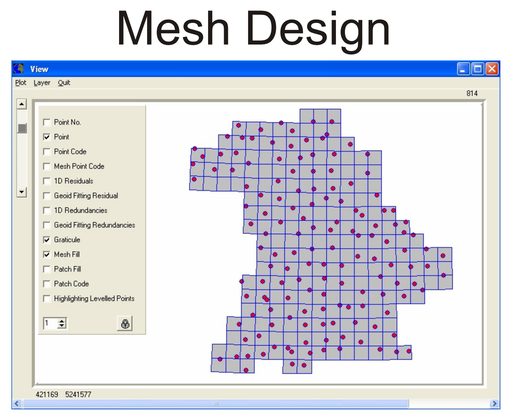
HRS Software



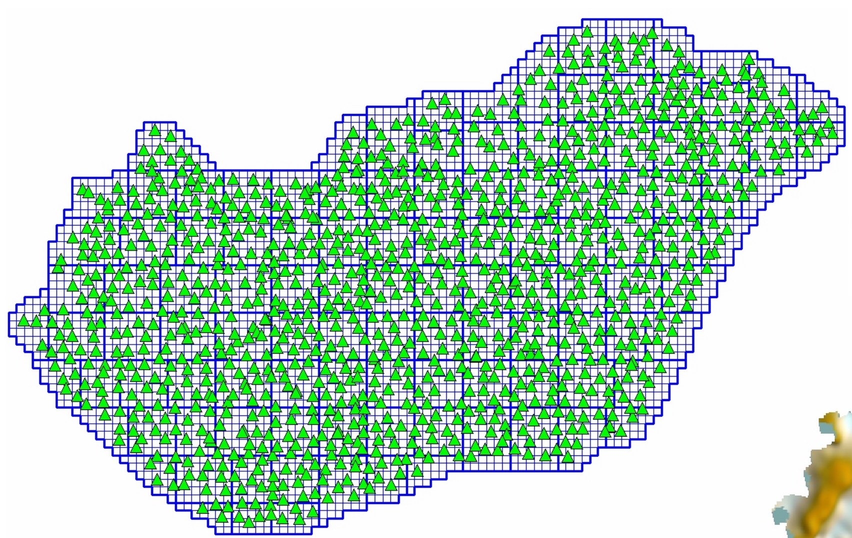
Histogram Construction



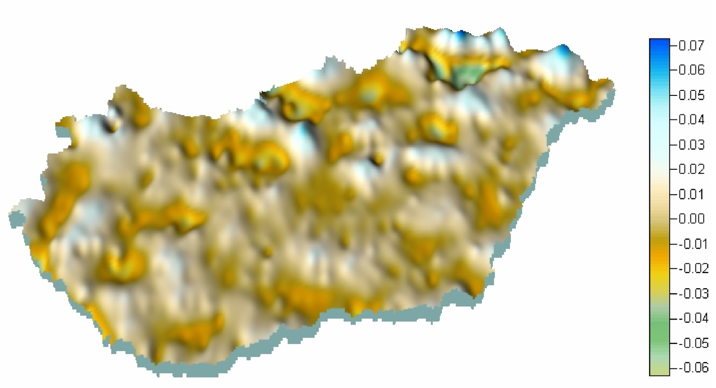
Mesh Design



DFHRS of Hungary



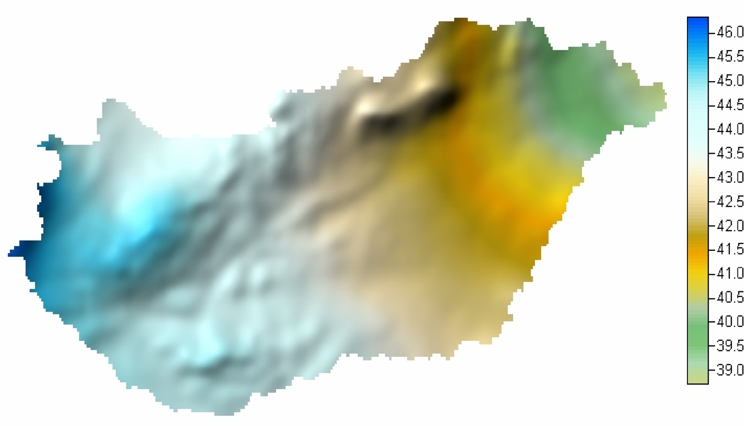
Geoid fitting residuals



Main characteristics

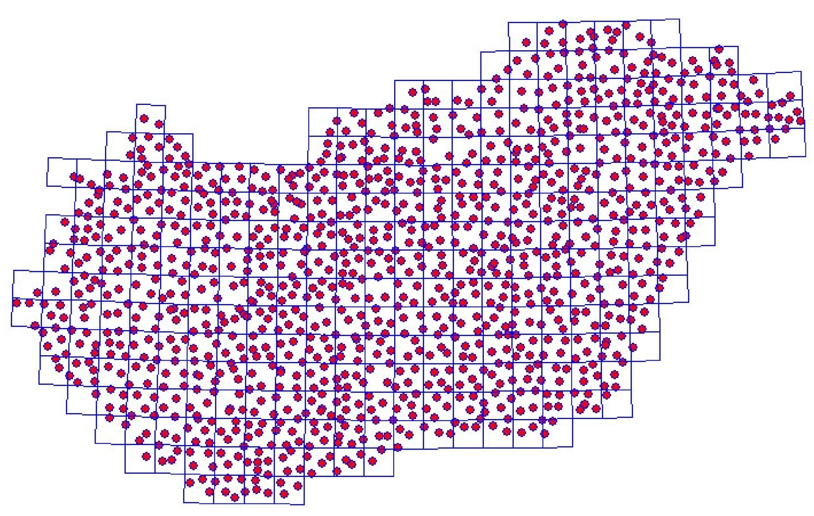
- 1150 fiducial points
- 5 km x 5 km mesh size
- Number of meshes = 4177
- Number of patches = 115
- EGG 97 geoid model
- Aposteriori standard deviations
  - ellipsoidal height fitting : 0.7 cm
  - height fitting : 0.9 cm

The geoid height map



www.dfhbf.de

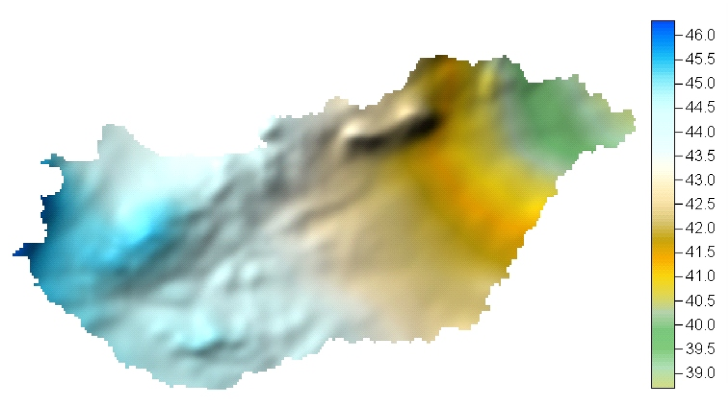
Three dimensional FEM HRS of Hungary



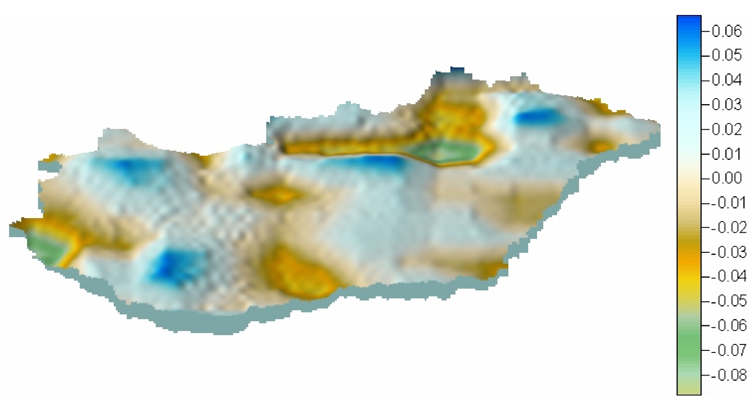
Main characteristics

- 1149 fiducial points
- 10' x 15' mesh size
- EGG 97 geoid model
- Aposteriori standard deviations
  - "h-H" fitting : 0.6 cm
  - "geoid model fitting" : 3.3 cm

The geoid height map



Geoid height differences from DFHRS solution



Data provided by Institute of Geodesy, Cartography and Remote Sensing, Hungary (FÖMI)