

Software development and its description for Geoid determination based on Spherical-Cap-Harmonics



Modelling using digital-zenith camera and gravimetric measurements hybrid data

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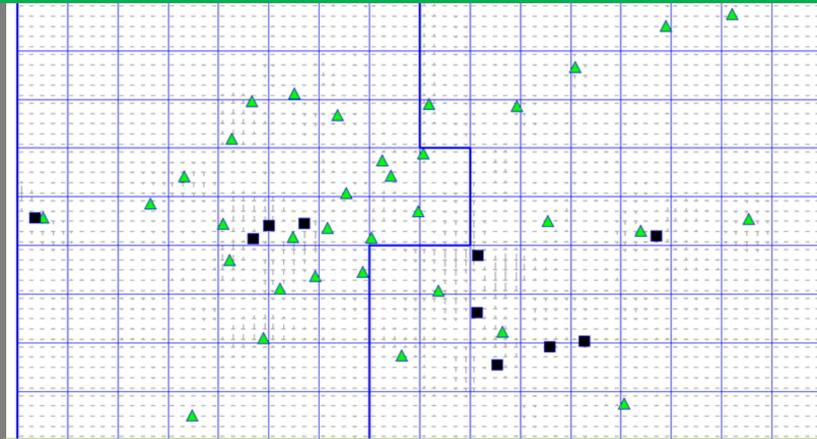
Acknowledgment

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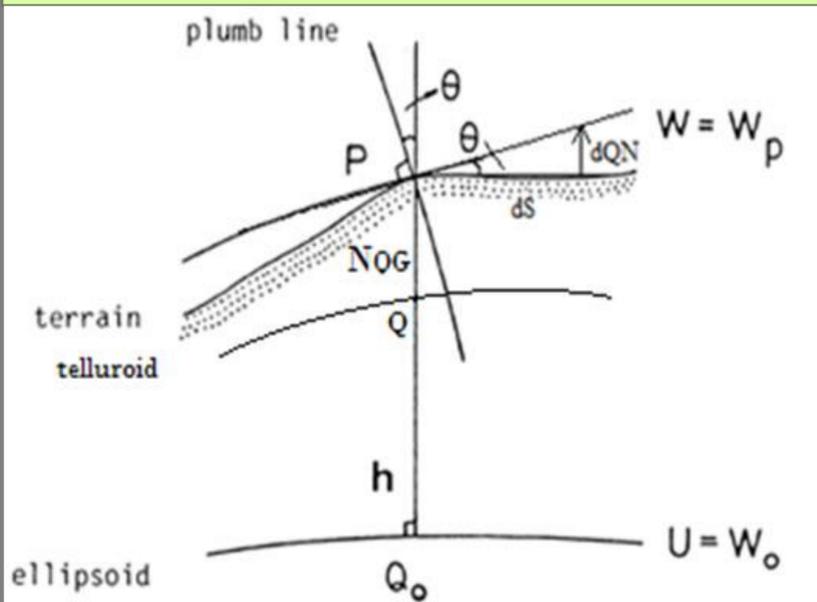


Introduction:

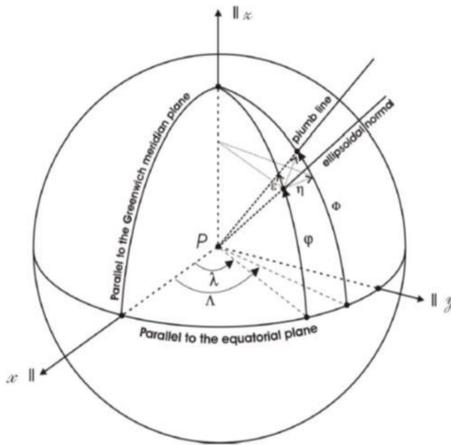
The DFHRS software has been developed at Karlsruhe University of Applied Sciences, Institute of Applied Research for the computation of precise quasi-geoid models from different kind of data starting in 1999 [1]. The computation of precise quasi-geoid models, providing the local quasi-geoid height  $N(B,L,h)$  as a function of the position in terms of the ellipsoidal geographical coordinates  $(B,L,h)$ , plays a significant role as geodetic infrastructure in the age of precise GNSS positioning technologies, in respect to transform by  $H=h-N$  an ellipsoidal GNSS height  $h$  into the physical normal height  $H$ . The GNSS based determination of physical heights  $H$  is much faster, easier to handle and much more economic, in comparison to classical geodetic levelling. The principle of the DFHRS-approach and software version 4.3 (v. 4.3) [1] is based on the parametric model of  $N(B,L,h)$  as a continuous polynomial height reference surface (HRS). The access to the parametric HRS model is enabled by DFHRS\_DB data-bases and access-software, which allows the direct conversion of GNSS-heights  $h$  into physical standard heights  $H$  by  $H=h-N$ . The DFHRS\_DB stores polynomial  $p$  parameters and (a scale-difference factor  $\Delta m$  for old height  $H$  systems) [1]. DFHRS v4.3 includes all types of geometrical input data: Both ellipsoidal heights  $h$ , and normal/orthometric heights  $H$ , geoid/Q-geoid-heights  $N$ , and deflections of the vertical  $(\eta, \xi)$  out of geopotential models (EGM2008) or grids. The updated software v4.3 includes observed vertical deflections data  $(\eta, \xi)$  from digital zenith cameras [2]. The region of Riga is chosen for tests of the camera system, data modelling tests and data quality analysis.



Riga region observations (green triangles – GNSS/levelling points, black squares – deflections of the vertical)



The relationship of unreduced deflections of vertical related to the Earth surface and the quasi-geoid



The astronomic latitude  $\Phi$  and longitude  $\Lambda$  determine the direction of the tangent to the plumb line, and the geodetic coordinates  $(B,L)=:(\phi, \lambda)$  define the direction of the ellipsoid normal. The deflections of the vertical  $(\eta, \xi)$  measured at the earth surface are the angular difference between plumb line direction and normal to the surface which consists of north and east component. The use of observations  $(\eta, \xi)$  measured by digital zenith camera allows to compute quasi-geoid model with much less GNSS/levelling points in a given area.

Vertical deflections  $(\eta, \xi)$ , here related to the sphere, and its components.

Further development of the software:

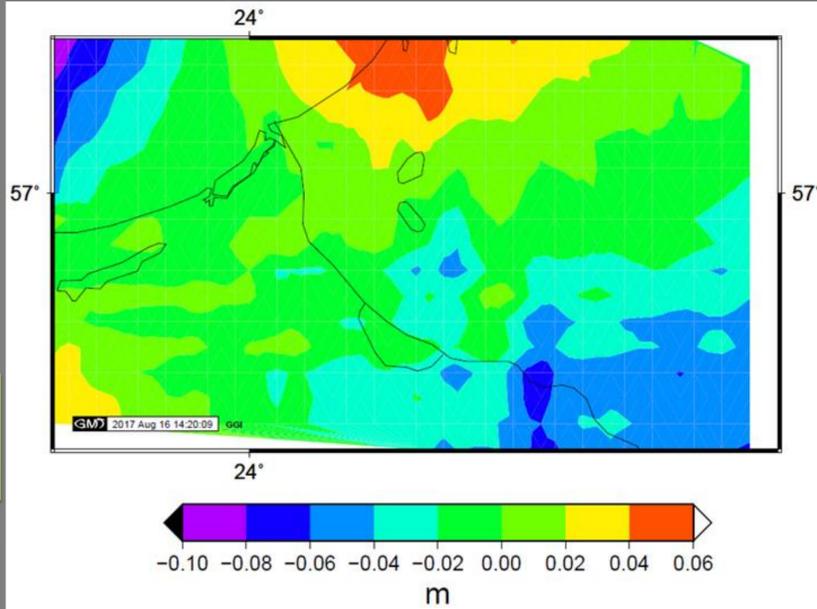
The advantage of spherical-cap-harmonics (SCH) modelling in comparison to spherical harmonics (SH) that less parameters are needed in order to compute local area instead of whole sphere. The gravitational potential  $V$  in terms of SCH for a point  $P(r, \alpha, \theta)$  within the cap reads:

$$V(r, \alpha, \theta) = \frac{GM}{R} \sum_{k=0}^{k_{max}} \left(\frac{R}{r}\right)^{n(k)} \sum_{m=0}^k (C'_{nm} \cos m\alpha + S'_{nm} \sin m\alpha) \bar{P}_{n(k),m}(\cos\theta)$$

Observation equations for the vertical deflections  $(\xi, \eta)$  at the Earth Surface  $P$ :

$$\xi_P = -\frac{\partial N_{QG}}{\partial B} \frac{\partial B}{\partial s_N} + \delta \xi_{norm.curv.} = -\frac{\partial B}{\partial s} \frac{\partial N_{QG}}{\partial B} + \delta \xi_{norm.curv.} = \frac{-1}{\gamma_Q(M+h)} \left(\frac{\partial T}{\partial B}\right)_P + \delta \xi_{norm.curv.}$$

$$\eta_P = -\frac{\partial L}{\partial s} \frac{\partial N_{QG}}{\partial L} = \frac{-1}{(N+h)\cos B} \frac{1}{\gamma_Q} \frac{\partial}{\partial L} T_P = \frac{-1}{\gamma_Q(N+h)\cos B} \left(\frac{\partial T}{\partial L}\right)_P$$



The impact of vertical deflections data from digital zenith camera

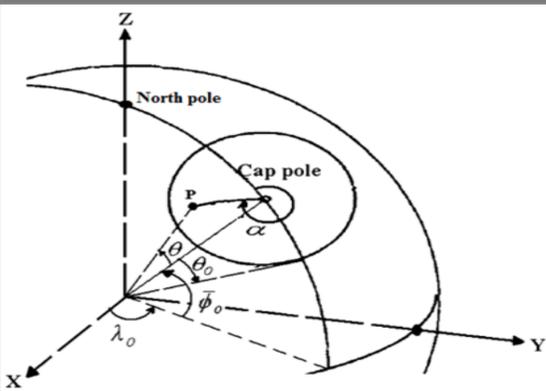
Conclusion:

The standard deviation of the vertical deflection data is equal to 0,09 arcsec for  $\xi$  (North-South) component and 0,14 arcsec for  $\eta$  (East-West) component, correspondingly.

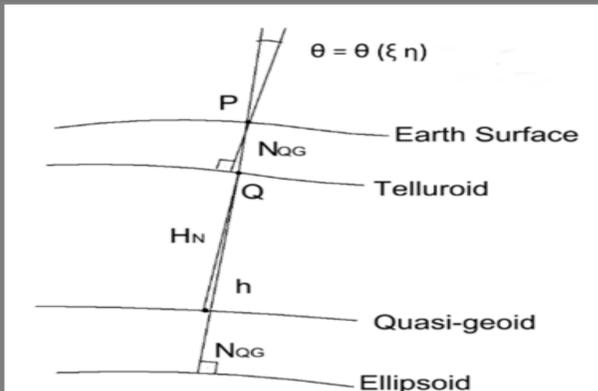
The calculations based on the preliminary results of vertical deflection observations prove the successful use of digital zenith camera and instrument readiness for further collection of observations. The computations using the DFHRS software v.4.3 allowed to carry out additional control and software's check for modelling and data errors in the frame of the data processing.

References:

- [1] [www.dfhbf.de](http://www.dfhbf.de)
- [2] Zariņš A et. Al 2016 Digital zenith camera of the University of Latvia, Geodesy and Cartography, 42:4, 129-135



Spherical cap area with its own pole located at the origin of area of interest



Deflection of vertical at point P