STATE OF THE ART AND PRESENT DEVELOPMENTS OF A GENERAL CONCEPT FOR GPS-BASED HEIGHT DETERMINATION

by

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ABSTRACT

The contribution treats a sophisticated concept in the area of GPS-based height determination with components being appropriate to branch out into different classes of standard approaches, depending on the kind of data sources as well as on the principal target. Basicly any kind of height data, namely geoid models N, vertical deflections (ξ,η), heights H, levelling ΔH, GPS heights h and GPS baselines Δh may be combined. A powerful mathematical tool within the general concept is the developed finite element model (FEM) surface approximation, which may be set up in different ways for the representation of the height reference surfaces (HRS). This FEM is parametrized by sets of bivariate polynomials, and continuity conditions guarantee a continuous transition of the FEM surface along the edges of neighbouring meshes in any area size. In opposite to digital terrain models, the nodes of the FEM mesh may differ from the position of the data used for the FEM determination.

The first part of the contribution treats the class of already practical working standard approaches, developed to transform in a statistically controlled way ellipsoidal GPS heights h into heights H of a standard height system. The so called "geoid refinement approach" as general standard means, that a datum adapted geoid model N is used as direct observation, while the above mentioned FEM serves as additional overlay to improve the final representation of the HRS. Together with the special cases of a "pure FEM approach" and a "pure geoid approach", all three approaches provide a flexible set of models, which are implemented in the software HEIDI2. Different pilot projects in several parts of Europe finished successfully, and the GPS height integration approaches are meanwhile used as a standard in several state survey departments. The experience shows that a high precision level for a GPS based height determination up to a 5 mm level in rather large areas is achieved, e.g. using the European gravimetric geoid (EGG97, see Denker and Torge, 1997).

A brief second part and class of approaches treats the application of the FEM component for the purpose of height system transformation (e.g. the conversion of so called NN-heights to normal heights started in Germany presently).

The third part of the presentation and class of approaches considers the so called general approach, where the HRS is completely established by a FEM, using different datum adapted geoid models N_G, terrestrial height information H and ellipsoidal GPS heights h as data sources. The result of the computation and "geoid mapping" respectively, leads to a <u>D</u>igital <u>F</u>EM <u>H</u>eight <u>R</u>eference <u>S</u>urface (DFHRS). Additionally also deflections of the vertical (ξ,η) are treated as a new observation component of the concept. The DFHRS may be set up as data base for a datum free direct GPS-based online heighting in DGPS networks. First results of a pilot project in the German SAPOS® (Satellite Positioning Service of the German Landesvermessung) network are reported.

1. INTRODUCTION

The transformation of a geocentric cartesian position (x,y,z) determined from DGPS provides the plane position represented by the geographical latitude and longitude (B,L) and the ellipsoidal height h, all referring to the datum of the respective reference station(s) used in DGPS. Both (B,L) and h depend on the metric and shape of the reference ellipsoid (a = main axis, f = flattening) used in the computation of (B,L,h). In general the GRS80 or the WGS84 ellipsoid are used in the context with GPS. The transition of GPS results $(B,L,h)_1$, in the following described as system 1, to a set of national network coordinates $(B,L,h)_2$, described as system 2, is to be performed by a threedimensional similarity transformation. There three translations (u,v,w), three rotations (e_x,e_y,e_z) and one scale difference Δm between both datum systems have to be taken into account. Using a taylor series expansion with linearization point $(B,L,h)_1$ and assuming small rotations, we may write the datum transition from system 1 to system 2 on splitting the threedimensional problem equivalently into the plane and the height component in the following way (Heiskanen and Moritz, 1967; Vanicek and Krakiwsky, 1986; Dinter et al., 1997a; Jäger 1998, 1999):

Plan components (1), (2) of the threedimensional datum transition

$$
\begin{aligned}
B_2 &= B_1 + \partial B_1(d) = B_1 + \partial B_1(u,v,w,e_x,e_y,e_z,\Delta m,\Delta a,\Delta f) \\
&= B_1 + \left[\frac{-\cos(L)\cdot\sin(B)}{M+h}\right]_1 \cdot u + \left[\frac{-\sin(L)\cdot\sin(B)}{M+h}\right]_1 \cdot v + \left[\frac{\cos(B)}{M+h}\right]_1 \cdot w + \\
&\quad \left[\sin(L)\cdot\frac{h+N\cdot W^2}{M+h}\right]_1 \cdot e_x + \left[-\cos(L)\frac{h+N\cdot W^2}{M+h}\right]_1 \cdot e_y + [0]\cdot e_z + \\
&\quad \left[\frac{-e^2\cdot N\cdot\cos(B)\cdot\sin(B)}{M+h}\right]_1 \cdot \Delta m + \left[\frac{N\cdot e^2\cdot\cos(B)\cdot\sin(B)}{a\cdot(M+h)}\right]_1 \cdot \Delta a + \left[\frac{M\cdot\sin(B)\cdot\cos(B)\cdot(W^2+1)}{(M+h)\cdot(1-f)}\right]_1 \cdot \Delta f
\end{aligned}
$$

$$
\begin{aligned}
L_2 &= L_1 + \partial L_1(d) = L_1 + \partial L_1(u,v,w,e_x,e_y,e_z,\Delta a,\Delta f) \\
&= L_1 + \left[\frac{-\sin(L)}{(N+h)\cdot\cos(B)}\right]_1 \cdot u + \left[\frac{\cos(L)}{(N+h)\cdot\cos(B)}\right]_1 \cdot v + [0]\cdot w + \\
&\quad \left[\frac{-(h+(1-e^2)\cdot N}{(N+h)\cdot\cos(B)}\cdot\cos(L)\cdot\sin(B)\right]_1 \cdot e_x + \left[\frac{-(h+(1-e^2)\cdot N)}{(N+h)\cdot\cos(B)}\cdot\sin(L)\cdot\sin(B)\right]_1 \cdot e_y + [1]\cdot e_z + \\
&\quad [0]\cdot\Delta m + [0]\cdot\Delta a + [0]\cdot\Delta f
\end{aligned}
$$

Ellipsoidal height component (3) of a threedimensional datum transition

$$
\begin{aligned}
h_2 &= h_1 + \partial h_1(d) = h_1 + \partial h_1(u,v,w,e_x,e_y,e_z,\Delta m,\Delta a,\Delta f) \\
&= h_1 + [\cos(L)\cdot\cos(B)]_1 \cdot u + [\cos(B)\cdot\sin(L)]_1 \cdot v + [\sin(B)]_1 \cdot w + \\
&\quad [e^2\cdot N\cdot\sin(B)\cdot\cos(B)\cdot\sin(L)]_1 \cdot e_x + [-e^2\cdot N\cdot\sin(B)\cdot\cos(B)\cdot\cos(L)]_1 \cdot e_y + [0]\cdot e_z + \\
&\quad [h+W^2\cdot N]_1 \cdot \Delta m + \left[\frac{-N\cdot W^2}{a}\right]_1 \cdot \Delta a + \left[\frac{W^2\cdot M\cdot\sin^2(B)}{1-f}\right]_1 \cdot \Delta f
\end{aligned}
$$

N(B) and M(B) are introduced as the latitude dependent quantities of the so called normal and the meridian radius of curvature respectively. For W(B) and e^2 we have W(B)=a/N(B) and $e^2=2f-f^2$. In general the parameter changes Δa and Δf are known, and the respective quantities are introduced as deterministic corrections. Respective corrections due to Δa and Δf are therefore not mentioned in the following.

It is worth to be mentioned, that the transformation of GPS-results $(B,L,h)_1$ into any other second plan system $(B,L)_2$ may be restricted to (1), (2) on using only plan coordinates $(B,L)_2$ as identical points for the determination of the datum parameters $\mathbf{d}=(u,v,w,e_x,e_y,e_z, \Delta m)$. In this way we may remain strict, and at the same time we need neither heights H_2 nor geoid information N_G (see fig. 1) from the national network height system 2. This plan integration approach was successfully applied e.g. in the new ITRF-related national network of Namibia (Christmann et al., 1999) and in other projects.

If we now go towards the problem of a GPS-based determination of standard heights H_2 referring to a physically defined height reference system HRS (fig. 1), we recognize, that because of $H_2=h_2-N_G$, we need some kind of "geoid model" N_G.
In this context we also have to consider in real life applications, that any geoid model N_G' taken from a geoid data base (Denker and Torge, 1997) has – for being just another surface in space – an own more or less small but unknown datum, which is to be described by a set of geoid datum parameters $\mathbf{d}_G=(u,v,w,e_x,e_y,\Delta m_G)$.

The principal relation between the standard height H_2, the ellipsoidal height h_2 and the height reference surface N_G of the height system in target of a GPS-based height determination reads

$$H_2 = h_2 - N_G \tag{4a}$$

and is shown in fig. 1. GPS provides the heights h_1 with respect to formulas (1), (2), (3). Due to the occuring datum effect $\mathbf{d}_G$ on using a geoid data-base $N_G(B,L)'$, we arrive in real life practice starting from (3) and relate it to (4a), at the complete formula reading:

$$H_2 = h_1 + \partial h_1(u,v,w,e_x,e_y,\Delta m) - (N_G' + \partial N(u,v,w,e_x,e_y,\Delta m)_G) = h_1 + \partial h_1(\mathbf{d}) - (N_G' + \partial N(\mathbf{d_G})) \tag{4b}$$

Assuming that the data base geoid values $N_G(B,L)'$ are referred to $(B,L)_1$, we see directly from (1), (2), (3) in the context with (4b), that the parameters within the different sets $\mathbf{d}$ and $\mathbf{d}_G$ separate due to the variation of the heights h_1 and N_G within the coefficients belonging to $\mathbf{d}$ and $\mathbf{d}_G$ respectively .

The standard approach of a threedimensional transformation, using only one common set of seven parameters $\mathbf{d}$, will therefore not be free from systematic errors. From (1), (2) and (4b) follows that a threedimensional GPS-integration based on standard heights H_2 and a geoid model N_G has to consider in total 13 parameters within $\mathbf{d}$ and $\mathbf{d}_G$.

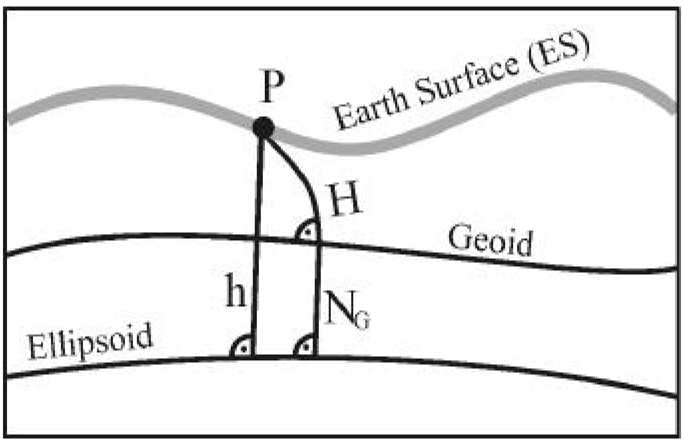

Figure 1. Ellipsoidal GPS height h, height reference surface HRS or briefly „geoid“ N_G and earth surface ES at a point P(B,L).

If we - in analogy to the plane problem discussed above – however restrict ourselves to a GPS-integration concerning the isolated GPS height integration problem, meaning the transformation of GPS heights h_1 to standard heights H_2, we derive from (3) and (4b) :

$$\begin{aligned} H_2 &= h_1 - N_G' + [\cos(L)\cdot\cos(B)]_1 (u-u_G) + [\cos(B)\cdot\sin(L)]_1\cdot(v-v_G) + [\sin(B)]_1\cdot(w-w_G) + \\ &\quad [e^2\cdot N\cdot\sin(B)\cdot\cos(B)\cdot\sin(L)]_1\cdot(e_x-e_{x,G}) + [-e^2\cdot N\cdot\sin(B)\cdot\cos(B)\cdot\cos(L)]_1\cdot(e_y-e_{y,G}) + \\ &\quad [h_1 + W^2\cdot N]\cdot\Delta m - [N_G + W^2\cdot N]\cdot\Delta m_G \\ &= h_1 - N_G + \partial_{h_1,G}(u',v',w',e_x',e_y') + [h_1 + W^2\cdot N]\cdot\Delta m - [N_G + W^2\cdot N]\cdot\Delta m_G \end{aligned} \quad (5a)$$

We recognize from (5a), that due to some common coefficients, only one set of common parameter-differences for the translations and rotations

$$\mathbf{d}' = (u', v', w', e_x', e_y') = (u-u_G, v-v_G, w-w_G, e_x-e_{x,G}, e_y-e_{y,G}) \quad (5b)$$

may be introduced, instead of two different sets, in a geoid-model based GPS-height integration. Separate parameters however have to be kept concerning the scales Δm and Δm_G in **d** and $\mathbf{d}_G$ (4b).

2. STANDARDS OF GPS HEIGHT INTEGRATION

With the trend of replacing old national datum systems in favour of ITRF-related datum systems and respective DGPS reference station systems (like e.g. SAPOS in Germany), the datum problem for the plan component (B,L) in GPS-based positioning will vanish by and by. But for the reason of a physically different height reference surface HRS for the standard heights H (fig. 1) defined by geopotential numbers, the problem of a transition of ellipsoidal GPS-heights h_1 to the standard heights H_2 referring to a HRS – or briefly spoken „geoid“ (a classical geoid for an orthometric height system, a quasi-geoid for a normal height system etc.) – will remain .

Different approaches have been developed by the "Karlsruhe working group" (Dinter et al., 1997a,b) up to now. The advantages of the above splitting into the plan (1), (2) and height component (3), (5a,b) respectively led to a powerful and flexible set of GPS-integration approaches, which will be presented and discussed in the following chapters 2.1, 2.2 and 2.3.

2.1. Finite Element Representation (NFEM) of the Height Reference Surface (HRS)

A powerful and central tool within the GPS height integration approaches of the "Karlsruhe working group" consists in the representation of the "geoid" N_G or better the height reference surface HRS (fig. 1) by a finite element surface, which is carried by the base functions of bivariate polynomials set up in the meshes of a square grid (fig. 2) with irregular nodal point positions (Dinter et al., 1997a,b).

In this way the finite element model NFEM(**p,**x(B,L),y(B,L)) of the HRS (6) represents in the ideal sense $h=H+N_G$ - datum free and independent of the type of the standard height system H - the height N_G of the HRS over the ellipsoid as a function of the plane position (B,L) and the parameter vector **p**. The plan position (B,L) is replaced in (6) by the metric coordinates (y(B,L)= “East“ and x(B,L)= “North“) such as UTM or Gauß-Krüger coordinates, which are functions of (B,L).

The mesh size and shape (fig.2), and at the same time the approximation quality of NFEM(**p**,x,y) with respect to the true HRS (fig. 1) may be chosen arbitrary. A special advantage and characteristic of the NFEM(**p**,x,y) representation consists last but not least in the fact, that the nodal points (•, fig. 2) of the FEM grid are totally independent of the geodetic network and data points (h,H,ΔH,Δh, the geoid heights $N_G(B,L)'$ and the vertical deflections (ξ,η)), which are used for the determination of the parameter vector **p** of NFEM(**p**,x,y). Without loss of generality we choose in the following bivariate polynomials of degree l as basic function to carry the surface NFEM(**p**,x,y) within the different meshes. The corresponding polynomial coefficients are introduced as $a_{ij,k}$, so that the total parameter vector **p** (6) consists of all coefficient sets $a_{ij,k}$ over m meshes (i=0,l; j=0,l and k=1,m).

$$\mathrm{NFEM}(p) =: \left\{ \begin{matrix} N(p_k) = \sum_{i=0}^{l} \sum_{j=0}^{l-i} a_{ij,k} \cdot y^i \cdot x^j \\ C_{0,1,2}(p_m, p_n) \end{matrix} \right\} \qquad (6)$$

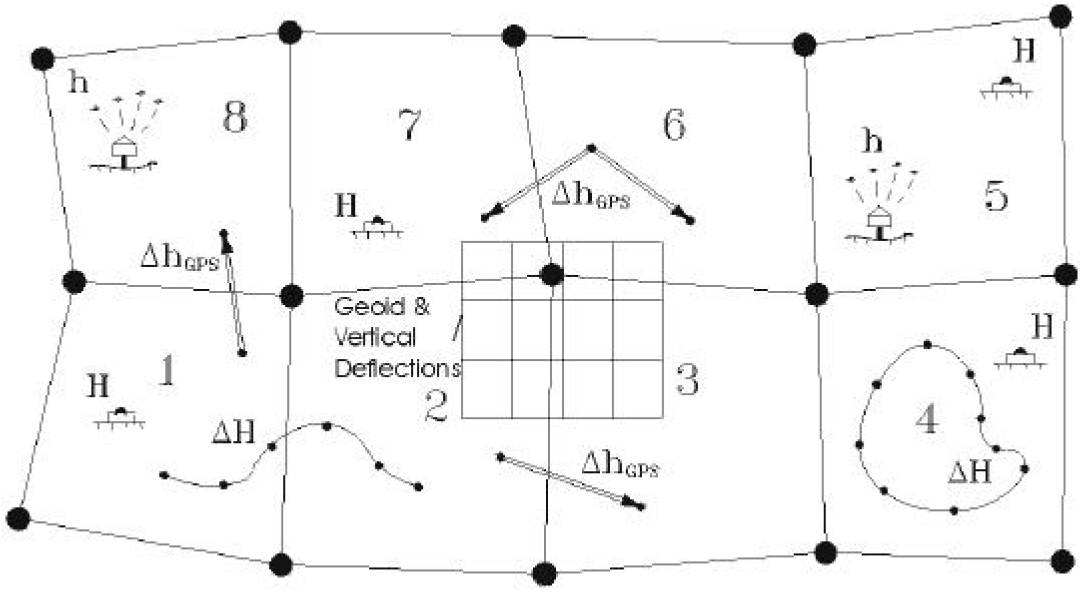

Fig. 2: Nodal points • and edges of a FEM-meshing and geodetic measurements (h, H, ΔH, Δh and optionally geoid-models N_G' and deflections of the vertical (ξ,η) as additional data sources, chap. 4.2.) for the determination of the finite element model NFEM(p,x,y) of the HRS or for geoid refinement.

Depending on the plan position (x,y) the local geoid height N_G is to be received from the finite element representation NFEM(**p**,x,y) by first identifying the corresponding k-th mesh according to the position (x,y) by means of the vector of nodal point positions. Then N_G is to be evaluated from NFEM(**p**,x,y) by the local polynomial with coefficients $a_{ij,k}$ at the plane position (x,y).

To imply a continuous surface NFEM(**p**,x,y) one set of continuity conditions of different type $C_{0,1,2}$ (6) has to be set up at the computation of NFEM(**p**,x,y) for each couple of neighbouring meshes m and n. The continuity type C_0 implies the same functional values along each common mesh border. The continuity type C_1 implies the same tangential planes and the continuity type C_2 the same curvature along the common borders of the HRS model NFEM(**p**,x,y). The continuity conditions occur as additional condition equations related to the polynomial sets of the coefficients $a_{ij,m}$ and $a_{ij,n}$ of each couple of neighbouring meshes m and n. The number and the mathematical contents of these condition equations depend on the polynomial degree l as well as on the continuity equation type (Dinter et al., 1997a,b).

The standard in the application of NFEM(**p**,x,y) in GPS height integration research and projects led to the best experiences on using C_0-conditions and a degree of l=1,2 for a small mesh sizes up to (10-40) km, and degrees up to l=3 for larger mesh sizes. For the case l=3 and C_o-continuity for NFEM(**p**,x,y) we have to introduce for each neighbouring mesh border the following condition equations (Dinter et al., 1997a,b):

$$da_{30}\, dx^3 + da_{21}\, dx^2dy + da_{12}\, dxdy^2 + da_{03}dy^3 = 0 \tag{7a}$$

$$da_{30}\Delta^3 + da_{20}\Delta^2dy + da_{10}\Delta dy^2 + da_{00}dy^3 = 0 \tag{7b}$$

$$da_{10}dxdy^2 + da_{01}dy^3 + 2da_{20}\Delta dxdy + da_{11}\Delta dy^2 + 3da_{30}\Delta^2dx + da_{12}\Delta^2dy = 0 \tag{7c}$$

$$da_{20}dx^2dy + da_{11}dxdy^2 + da_{02}dy^3 + 3da_{30}\Delta dx^2 + 2da_{21}\Delta dxdy + da_{12}\Delta dy^2 = 0 \tag{7d}$$

With respect to the known nodal points $A(y_A,x_A)$ and $E(y_E,x_E)$ of the mesh grid (fig. 2) we introduced in (7a,b,c,d) the abbreviations $dx=x_E-x_A$, $dy=y_E-y_A$, $\Delta=dy\cdot x_E-dx\cdot y_A$ and $da_{ij}=a_{ij,m}-a_{ij,n}$. The index A means the starting and the index E the ending point of the common border of each two neighbouring meshes m und n.

2.2. Standard approaches of GPS height integration

Starting with formula (5a,b) we immediately arrive at the so called pure geoid approach. This approach is to be applied in a GPS height integration, as soon as good geoid information $N_G(B,L)'$ is available. The parameters within the datum part $\partial_{h,G}(\mathbf{d'},\Delta m_G)$ have to be estimated. With some simplification in the scale term[1] for Δm, the pure geoid approach reads in the corresponding system of observation equations as follows:

$$h + v = m\cdot H + N_G \tag{8a}$$

$$N_G\,(B,L)' + v = N_G + \partial_{h,G}\,(\mathbf{d'},\Delta m_G) \tag{8b}$$

$$H + v = H \tag{8c}$$

GPS heights h, a geoid model $N_G(B,L)'$ and terrestrial heights H may be used as obervations. Of course the formulas are easy to extend to levelling ΔH and GPS height baselines Δh, which are also both included in all subsequent approaches. Apart from the datum part $\partial_{h,G}\,(\mathbf{d'},\Delta m_G)$ the geoid model $N_G(B,L)'$ is treated as so called ‚direct observation‘. The datum part $\partial_{h,G}\,(\mathbf{d'},\Delta m_G)$ (see also fig. 3) may also model and remove some long waved systematics between the geoid model $N_G(B,L)'$ and the HRS (Dinter et al., 1997a,b; Dinter, 1997).

In polarity to (8a,b,c) and for case respectively, that no geoid-information is available, we may derive the HRS completely from the observations (h, H, ΔH, Δh) as a finite element representation NFEM(**p**,x,y) (6) of the HRS. This approach is called the pure finite element approach. It reads:

$$h + v = m\cdot H + \mathrm{NFEM}(\mathbf{p},x,y) \tag{9a}$$

$$H + v = H \tag{9b}$$

The powerful synergy of both above approaches finally leads to the so called „geoid-refinement approach“. It is used for the case that the available geoid information $N_G(B,L)'$ is to be refined by a finite element model NFEM(**p**,x,y), which is acting as additional overlay to improve the geoid model (see fig. 3). The geoid-refinement approach reads:

---

[1] The strict scale term according (3) looks like in (12a).

$$h \;+\; v \;=\; m \cdot H + N_G \qquad (10a)$$
$$N_G(B,L)' + v = N_G + \partial_{h,G}(\mathbf{d}', \Delta m_G) + NFEM(\mathbf{p},x,y) \qquad (10b)$$
$$H \;+ v \;=\; H \qquad (10c)$$

The pure geoid aproach (8a,b,c) and the pure finite element approach (9a,b) are resulting as special cases of the above geoid-refinement approach (10a,b,c).

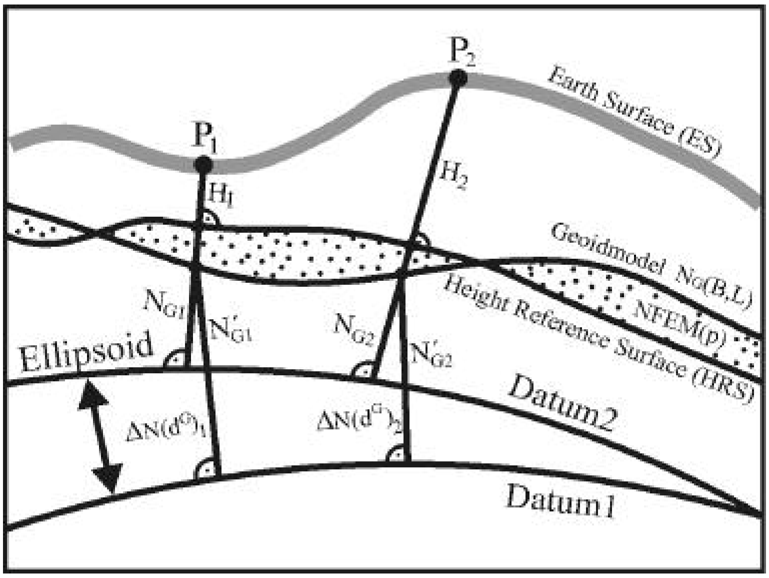

Figure 3. Geoid-refinement approach as a synergetic combination of geoid information $N_G(B,L)'$ submitted to a datum change (N_G', datum 1 → N_G, correct datum 2) and the finite element model NFEM(p,x,y) as overlay (dotted). It is introduced to describe remaining systematics between N_G and the true height reference surface HRS .

2.3. Example of a GPS height integration performed with the software HEIDI2

The following example of a GPS height integration treats the use of the commercially available EGG97 geoid model (Denker and Torge, 1997) for an integration of GPS heights h into the normal height system H of the height network of Tallinn, Estonia. The network has an extension of 40 km by 25 km. The computations were done by the author in the frame of a European so-called TEMPUS project between the Tallinn Technical University, the University of Technology Karlsruhe and other European universities. The given 23 ellipsoidal GPS-heights h in the EST92 datum were introduced with a standard deviation of 3 mm, as proved before in a free adjustment of the respective GPS height baselines.

The given normal heights were introduced with a standard deviation of 3mm, and the EGG97 observations $N_G(B,L)'$ with a precision of 5 mm. The different versions of the GPS height integration were computed on the base of the pure geoid approach (8a,b,c) with the software HEIDI2 © Dinter-Illner-Jäger.

The result of a first version, where - in sense of the unrealistic ideal (4a) - no datum transition $\partial_{h,G}(\mathbf{d}',\Delta m_G)$ for h and $N_G(B,L)'$ was introduced, is presented in fig. 4. Each known point was once computed as a „new point“ determined by „GPS and geoid“. The residuals in the identical points H are

in the range of up to ± 3.5 cm and show the typical effect of a neglected datum tilt in this high range (see also Dinter et al., 1997 a,b).

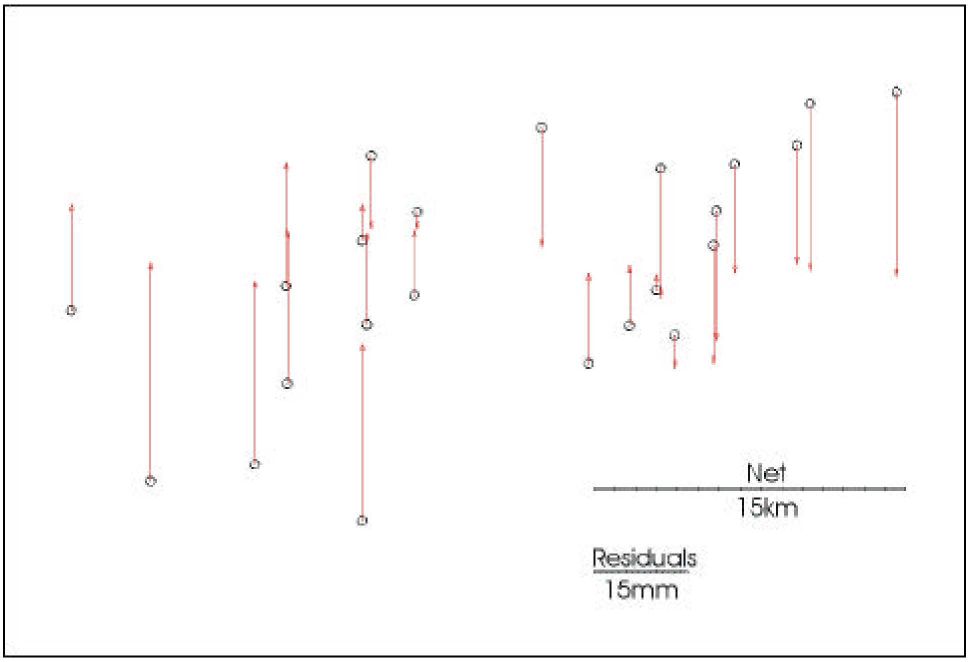

Figure 4: GPS height integration for the Tallinn network by the pure geoid approach without taking a necessary datum-transition $\partial_{h,G}$ (d‘,Δm_G) part for h and N_G(B,L)' into account: The residuals in the known control points - treated as new points - show the systematics of a datum tilt up to ± 3.5 cm.

The fig. 5 shows the next set of computations due to the pure geoid approach (8a,b,c) used as computation model for a GPS height integration. Now a datum transition $\partial_{h,G}$(**d‘**,Δm_G) (8b) for N_G(B,L)' was taken into account. The residuals in the known control remain less than 1 cm, the mean residual is in the range of ± 4 mm. In this version of a GPS-based height integration all observation components were consistent with their assumed a priori precision and no gross errors occured in all runs. An additional geoid refinement might be computed by the geoid refinement approach (10a,b,c).

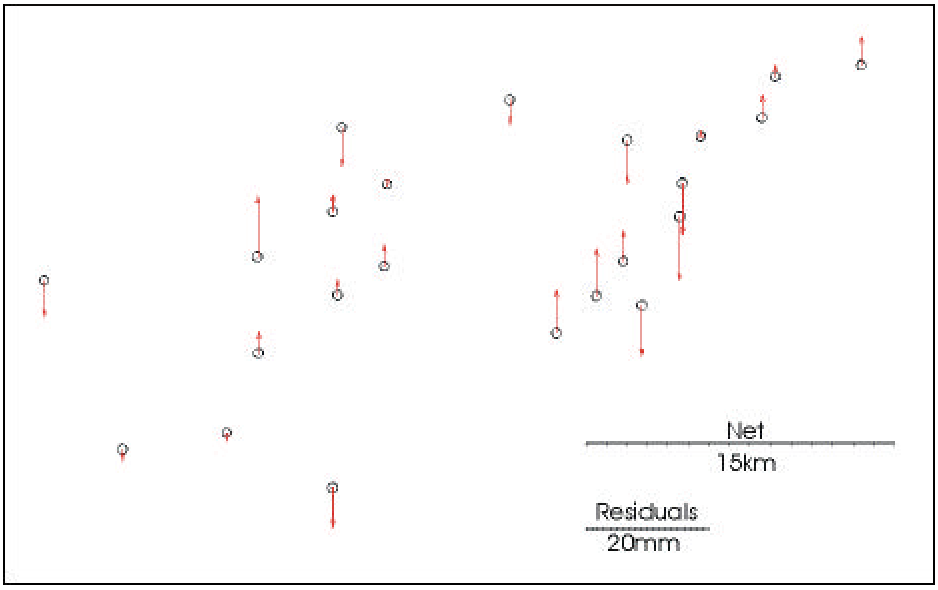

Figure 5. GPS height integration for the Tallinn network by the pure geoid approach on taking the necessary datum-transition $\partial_{h,G}$ (d‘,Δm_G) part for h and N_G(B,L)' into account. The residuals in the known control points - treated as new points – now keep in a mean range of only ± 4mm.

For further examples of GPS height integration in medium and in large scale networks and also due to the other above approaches like the geoid refinement approach (10a,b,c) and the pure FEM approach (9a,b) it is referred to (Dinter et al., 1997a,b; Dinter, 1997; Jäger und Mengesdorf, 1998; Jäger, 1999).

3. HEIGHT SYSTEM TRANSFORMATION

The essential components of the above GPS height integration concept - namely the datum transformation part for heights (3) and the finite element representation NFEM(**p**,x,y) of a HRS (6) - may be transferred to the problem of transforming old heights H_{old} to new heights H_{new} of a new height system. In analogy to the above geoid refinement approach (10a,b,c) the most general approach for a height system transformation reads:

$$H_{old} + v = H_{new} + \partial H(\mathbf{d}) + HFEM(\mathbf{p},x,y) \quad (11a)$$
$$H_{new} + v = H_{new} \quad (11b)$$

The datum transformation parameters **d** as well as the parameters **p** of the finite element model are to be determined by identical points (H_{old}, H_{new}) in both systems.

4. ONLINE GPS-HEIGHTING – PRODUCTION AND APPLICATION OF A DIGITAL FINITE ELEMENT HEIGHT REFERENCE SURFACE (DFHRS)

4.1. Digital Finite Element Height Reference Surface (DFHRS) concept for an online GPS Heighting

The profile and target of an online GPS-Heighting is easy to formulate (see fig. 6): An ellipsoidal GPS-height h, determined at a position P(y(B,L), x(B,L)), is to be made convertable directly to the height H of the standard height system. The converted height H should result online on applying a respective correction to h, and the resulting H should not suffer with a quality-decrease compared to the heights H resulting from a postprocessed GPS height integration (applying the approaches presented in chap. 2).

In this chapter a general concept is presented, which fulfils all above requests and shows besides this even some more positive aspects. The concept is to produce in a first step in a controlled way a so called **D**igital-**F**inite-**E**lement-**H**eight-**R**eference-**S**urface (DFHRS) as a new kind of data base product (= production step).

The second step is to make this data base accessible online – in an active or passive way - for DGPS- Heighting (= application step). That means, that either the DGPS user has the DFHRS at his disposal or the DGPS service exclusively uses the DFHRS for the evaluation of a correction Δ to transform a GPS height h to the height H of the standard height system (principle, see fig. 6).

The production step of the DFHRS reads in the system of observation equations as follows:

$$h + v = H - (h+N\cdot W^2)\cdot \Delta m + NFEM(\mathbf{p},x,y) \quad (12a)$$
$$N_G(B,L)' + v = NFEM(\mathbf{p},x,y) - \partial_{h,G}(\mathbf{d}', \Delta m_G) \quad (12b)$$
$$H + v = H \quad (12c)$$

Identical points (H, h), and if available, one or a number of geoid models $N_G(B,L)'$, are used as observations to produce the DFHRS by a least squares estimation related to (12a,b,c). The DFHRS on the right side is represented completely by the finite element model NFEM(**p**,x,y) of the HRS.

NFEM($\mathbf{p}$,x,y) is modelled according to (6) and again togehter with continuity conditions. This means that the geoid model input $N_G(B,L)'$ is "mapped" to the DFHRS by removing the datum part $\partial_{h,G}(\mathbf{d'},\Delta m_G)$. An additional NFEM-refinement term may be set up in (12b). The production step of the DFHRS (12a,b,c) has to be embedded in a statistical quality control concept, e.g. of a least squares estimation, so that any component including the input of „mapped“ and datum-adapted geoid-model, can be controlled (Ackermann, 1999; Schwarzer, 2000).

The decisive components and formula parts of the production step, which are afterwards needed in the application step – namely in an online GPS-Heighting - are contained in (12a). Equation (12a) leads to the following correction scheme, which has to be applied to the GPS height h in an online application of the DFHRS data base with respect to convert it to the standard height H:

$$H = h + \Delta = h + corr1 + corr2 = h - NFEM(\mathbf{p},x,y) + (h+N\cdot W^2)\cdot\Delta m \qquad (13)$$

The first correction part „corr1“ is due to the DFHRS („geoid correction“), and „corr2“ is due to the scale Δm between the GPS heights h and those of the standard height system H.

4.2. Extension of the approaches with respect to deflections of the vertical

As a pursuit of the DFHRS concept given in Jäger (1998, 1999) the central cernal of a "geoid mapping" related to the geoid heights $N_G(B,L)'$ is now extended to vertical deflections ξ (meridian) and η (prime vertical) as additional data sources for the evaluation of the DFHRS. The vertical deflections ξ and η may either result from astronomical observations or are just to be taken from any modern "geoid data base" (Denker and Torge, 1997). Starting with the interesting DFHRS-representation (6) we first rewrite NFEM($\mathbf{p}$,x,y) as the following product:

$$NFEM(\mathbf{p},x,y) = F(x(B,L),y(B,L)) \cdot \mathbf{p} \qquad (14a)$$

With the partial derivatives F_B and F_L of F(x,y) in latitude and longitude respectively, which depend on the individual type of the mapping functions x=x(B,L) and y=y(B,L), and the standard formulas for the differential way increment ds on the ellipsoid,

$$\partial s/\partial B = M(B) \text{ and } \partial s/\partial L = N(B)\cdot\cos(B) \qquad (14b)$$

we arrive at the following observation equations for vertical deflection observations ξ and η:

$$\xi + v = -\mathbf{F}_B / M(B) \cdot \mathbf{p} + \partial B(\mathbf{d}_{\xi,\eta}) \qquad (14c)$$
$$\eta + v = -\mathbf{F}_L/(N(B)\cdot\cos(B))\cdot \mathbf{p} + \partial L(\mathbf{d}_{\xi,\eta})\cdot\cos(B) \qquad (14d)$$

M(B) and N(B) are again the so-called meridian and normal radius of curvature respectively. The datum parts $\partial L(\mathbf{d}_{\xi,\eta})$ and $\partial L(\mathbf{d}_{\xi,\eta})$ are set up according to (1) and (2) with a datum parameter set $\mathbf{d}_{\xi,\eta}$. In case of vertical deflection observations (ξ,η) related to the geoid surface the "height" h must be set to h=: $N_G(B,L)'$ in $\partial L(\mathbf{d}_{\xi,\eta})$ and $\partial B(\mathbf{d}_{\xi,\eta})$.

It is evident that using (14c) and (14d) - which are in general available on geoid data bases (Denker and Torge, 1997), but remain unused in the context of practical GPS-integrations and most GPS-height integration concepts - will further increase the geometric quality and the reliability of respective DFHRS data base computations.

The observation equations (14c,d) may of course also be added to those belonging to the standard of a postprocessed GPS height integration (10a,b,c).

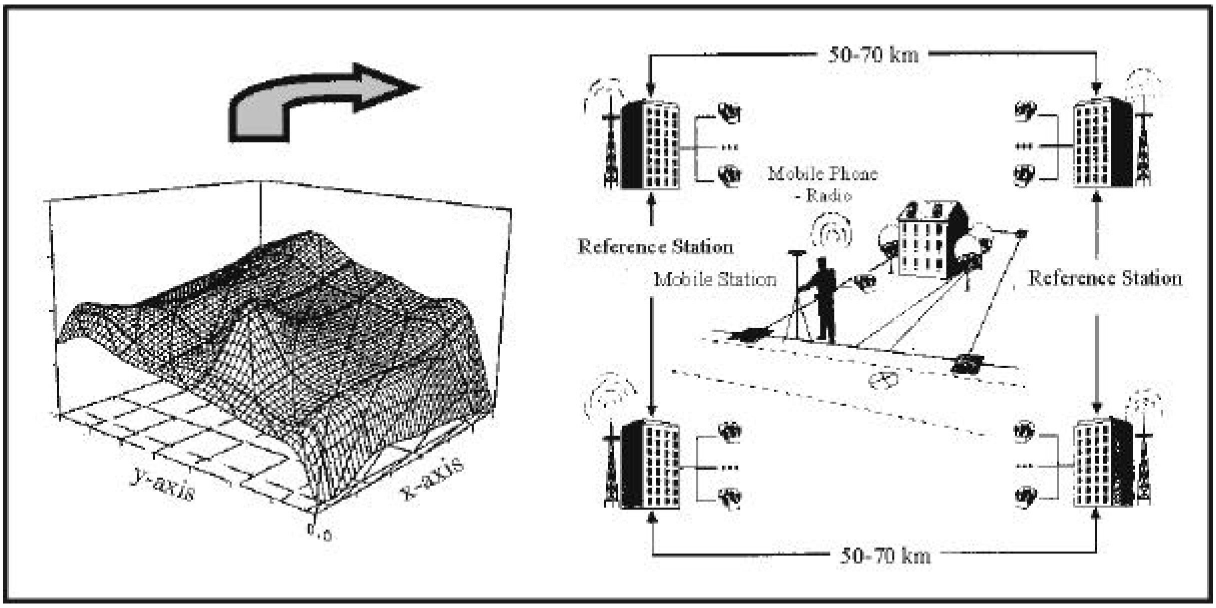

Figure 6. DFHRS (left) and its use (right) as DFHRS data base for a DGPS-based online heighting (h→H).

4.3. Example of a DFHRS production

Fig. 7 shows the finite element grid of the 30 km by 30 km "Mosbach" area near Heidelberg, where a DFHRS was computed restricted to (12a,b,c) in the frame of a pilot project (Ackermann, 1999). For different investigations the whole area was parted into 1, 4, 9 and at maximum 16 meshes as presented in fig. 7.

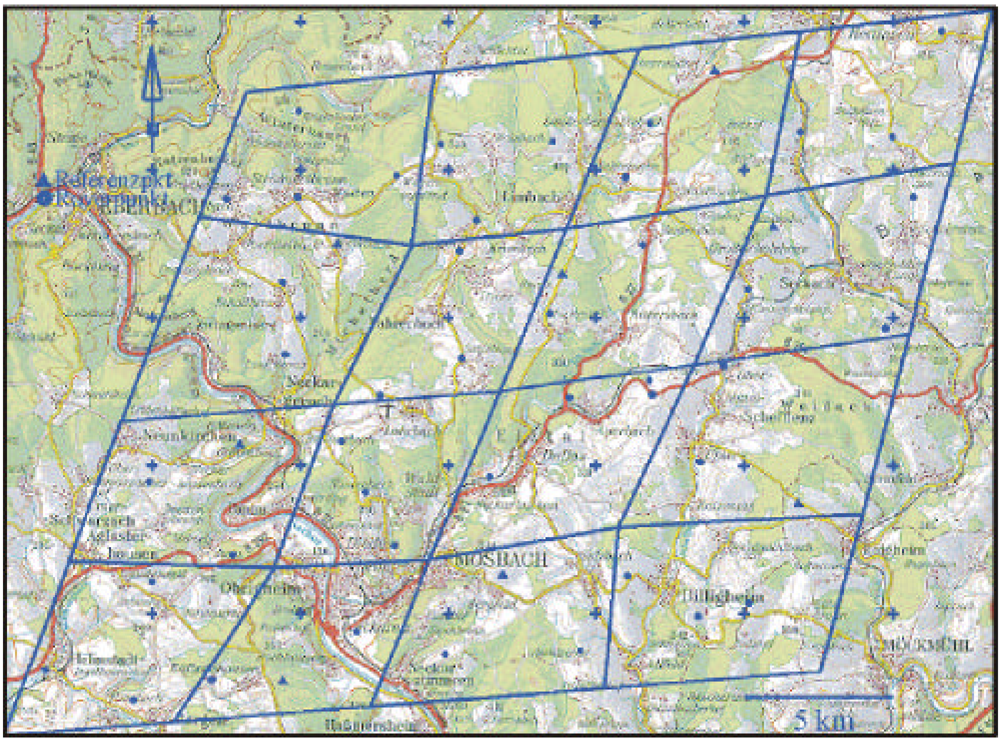

Figure 7. Meshing and data design of the identical points (H,h) of a DFHRS computation for the Mosbach region.

: **map grid, • GPS Rover Station, GPS Reference Station**

A first kind of DFHRS was produced using all 46 identical points (H, h) in both systems but no additional geoid information $N_G(B,L)'$. The best DFHRS resulted for this case by a polynomial degree l=2 and a k=4 meshes grid with an avarage side lenght of 12 km. In this way the complete DFHRS for the area (fig. 7) is represented by k=4 sets of each six coefficients $\mathbf{p}_k=(a_{oo}, a_{o1}, a_{1o}, a_{o2}, a_{11}, a_{2o})_k$ and an additional scale parameter both evaluated from (12a,c). The resulting DFHRS shows a mean accuracy of less than 1cm, and the maximum residual of the conversion of ellipsoidal GPS heights h to standard heights H on using the DFHRS and the respective correction (13) formula remains smaller than 1.5 cm.

About the same results of quality for the DFHRS were achieved using the complete "geoid mapping" as given in (12a,b,c). The computations were performed using the EGG97 as geoid model $N_B(B,L)'$ and again a polynomial degree l=2 and k=4 meshes. The advantage of the complete approach is of course, that the numer of identical points (h,H) is to be kept low on a minimum of 7 points (even only 4 in a simplified datum parametrization) (Ackermann, 1999; Dinter et al., 1997a,b) for the whole area, according to the number of datum parameters ($\mathbf{d}'$, Δm_G, Δm) (4b), (5a,b). No gross errors were detected in the observation components h, $N_B(B,L)'$ and H in the step of DFRHS-production (12a,b,c). Other successful computations were performed recently for the above Tallinn area (fig. 4, 5) and for a 60 km by 80 km area at Frankfurt in Hessen, Germany (Ackermann, 1999).

4.4. Outlook for the DFHRS concept

The DFHRS can be characterized as a new product appropriate for an online GPS-heighting with best quality and economical properties. The wellknown geoid datum problem and individual datum calibration steps using identical points (h,H) during or before GPS heighting respectively are completely dropped out. The DFHRS enables a direct GPS-heighting with a general suitability for anybody in the frame of DGPS-applications and DGPS services.

The production of the DFHRS (12a,b,c) is performed as an overdetermined least squares adjustment, which enables a quality control of all components including the input of geoid models $N_B(B,L)'$. The computation of the DFHRS product may be repeated at any time, as soon as new data (h, H, ΔH, Δh, $N_B(B,L)'$ and (ξ,η)) arise, or even if a change of the height system type H or height datum is intended.

The variation of the mesh size further enables to produce on demand different DFHRS products with a different geometric quality (and price) for an online height positioning purpose. Besides that there are two different ways for a DFHRS marketing: The first way is to keep the DFHRS on the side of the data- and DFHRS owner and transmit only the correction Δ (13) by the DGPS-service. This requires however that the DGPS customer transmits his GPS-position (B,L,h) to the DGPS-service and gets back the converted height value H. The other way is of course to sell - like also usual in the context with modern geoid-models (Denker and Torge, 1997) – the DFHRS data base directly to the DGPS user. The first experiences with the DFHRS concept based on (12a,b,c) are much promising (Ackermann, 1999).

As for most countries geoid information $N_G(B,L)'$ is available, the DFHRS evaluation may in general be set up together with a "geoid mapping" (12b). For the case (12a,b,c) the best quality and control of the DFHRS will be achieved, and at the same time the number of identical points (h,H) for control and datum parameter estimation remains small even for large areas. The development of comfortable and powerful C++ software for the production of DFHRS data bases is continued

(Schwarzer, 2000), and comprising also the implementation and exploitation of additional observations (14c,d) of type vertical deflections (ξ,η). So the observation amount in DFHRS computation related to a "geoid data mapping" - completed with respect to (ξ,η) - will be three times higher than on using only N_G(B,L)'. The additional vertical deflection observations (ξ,η) – i.e. taken from a geoid data base (Denker and Torge, 1997) - will at the same time provide an essential improvement of the geometric quality and an increase of the statistical reliabilty of a DFHRS.

The additional observation equations (ξ,η) (14c,d) may of course also be set up in a classical postprocessed GPS-height integration in addition to (10a,b,c).

Concerning the DFHRS concept, the implementation of software for the production step (Ackermann, 1999; Schwarzer, 2000) as well as for the application of resulting DFHRS data bases was in online DGPS carried out recently (Seiler , 1999).

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