DFHRS – A rigorous Approach for the Integrated Adjustment and Fitting of Height Reference Surfaces[1]

Status Report of the Research and Development Project DFHBF (www.dfhbf.de)

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1. Introduction

The DFHRS (Digital-Finite-Element-Height-Reference-Surface) research and development project, German word DFHBF, aims at the computation of height reference surfaces (HRS) [1]. A HRS is represented by the height N of the HRS above the reference ellipsoid (fig. 1). The main practical target of the DFHRS project is to enable by using Global Navigation Satellite Systems (GNSS), such as GPS and GLONASS or future ones like GALILEO and COMPASS, in GNSS-based positioning in reference station networks (e.g. SAPOS, www.sapos.de or ascos, www.ascos.de) the direct conversion of the ellipsoidal GNSS height h determined at the earth surface (ES), into the physical earth gravity field based standard "sea-level" height H = h – N (fig. 1).

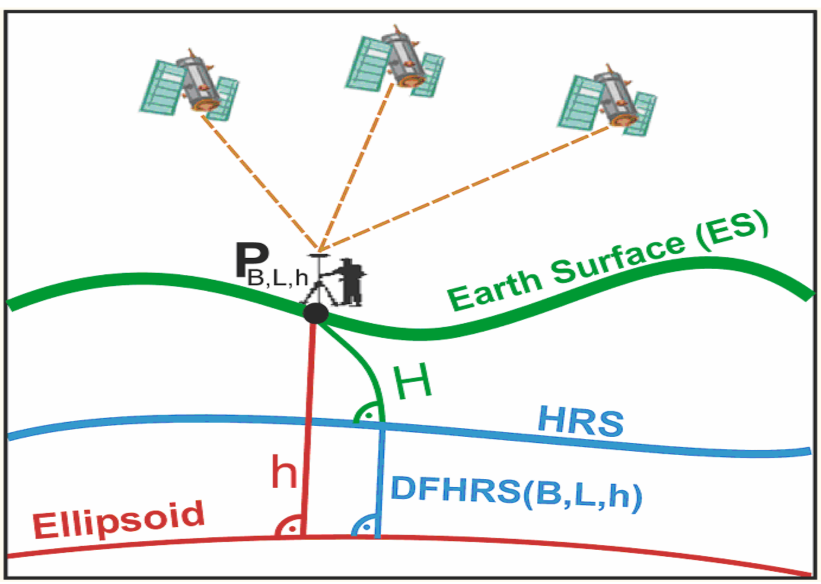


Fig. 1:
Principle of GNSS-based height determination: H = h - N = h – DFHRS(B,L,h)

Depending on the height system type, the physical heights H are called orthometric, normal or spheroidal normal heights (NN-heights), and the respective HRS is called geoid, quasi-geoid or NN-surface. In the DFHRS concept, a continuous polynomial surface over of a grid of finite element meshes (FEM) with polynomial parameters **p** (fig. 3, thin blue lines) is used as a carrier function N=N(B,L,h) for the HRS. The FEM surface of the HRS is therefore called NFEM(**p**|B,L). The above HRS-types show weak or missing dependences of the HRS height N from h, which is treated in the mathematical computation model of the DFHRS approach below, and therefore already included in the final HRS representation NFEM(**p**|B,L). For some old height systems H a scale-difference factor Δm has to be considered additionally, so that the DFHRS-correction DFHRS (fig. 1) consists of two parts. The

[1] Published on the DFHRS WebSite: www.dfhbf.de

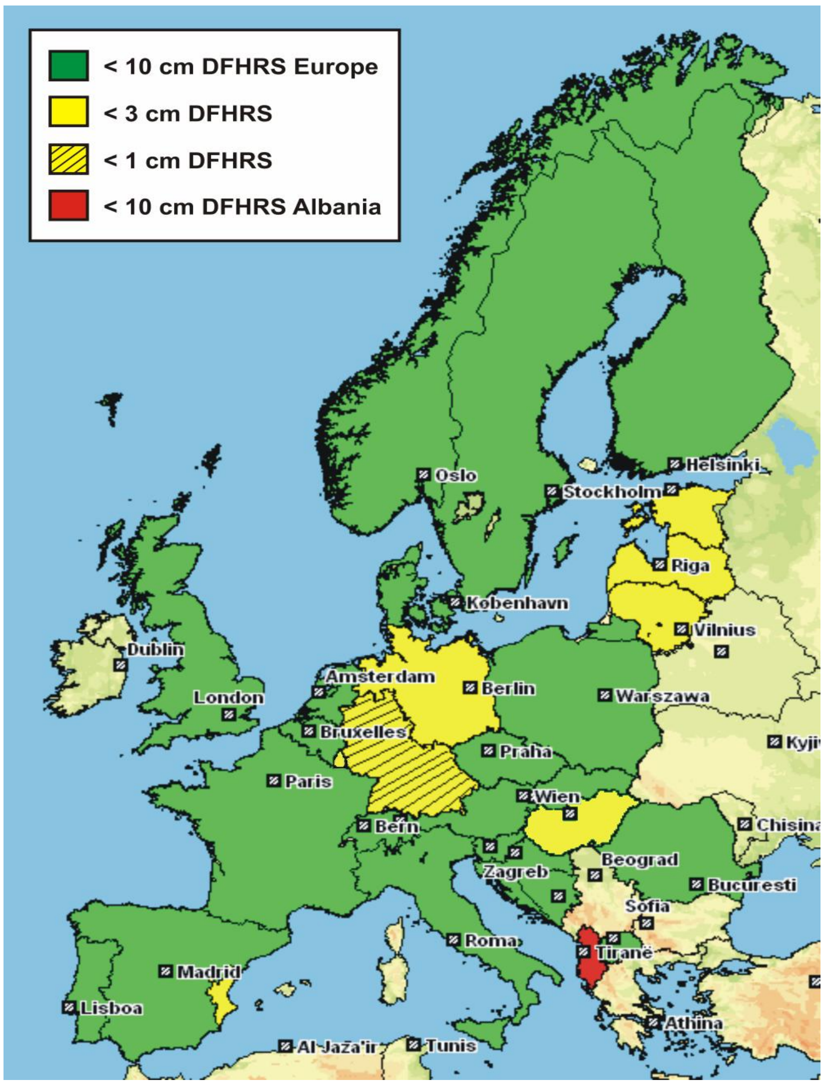
principle of a GNSS-based height determination H (fig. 1), requires to submit the GNSS-height h to the DFHRS(B,L,h)-correction, and it reads:

$$H = h - \mathrm{DFHRS}(p, \Delta m \mid B, L, h) = h - (\mathrm{NFEM}(p \mid B, L) + \Delta m \cdot h) \quad . \tag{1}$$

The DFHRS-correction DFHRS(B,L,h) is provided by means of a DFHRS database (DFHRS_DB), which contains the HRS parameters (**p,** Δm) together with the mesh-design (fig. 3) information. DFH-RS_DB have become an industrial and user standard for all GNSS-receiver types worldwide ([1], [3], [5], [7], [8]), and a new kind of modern geo-data product [4] in the GNSS navigation age.

2. Geometrical Observation Components and Parametrization – 1st stage of the DFHRS concept

In the 1st stage of the DFHRS approach development, geoid- or geopotential model (GPM) heights N, observed astronomical or geoid/GPM-model based deflections of the vertical (ξ,η) in any number of groups, and fitting points (B,L,h; H) were exclusively used as observation groups in a common least squares computation for the evaluation of the DFHRS_DB parameters **p** and Δm. The mathematical model for these observations is given by formulas (2a-f). In case of an adequate stochastical model, the use of gravity-based geoid-/GPM model information is equivalent to the use of the original observed gravity values g (see, [3]).



<u>Fig. 2: </u>Overview of DFHRS_DB computed all over Europe

The mathematical model for the computation of the DFHRS_DB parameters (**p,** Δm) using the above so-called geometrical part of observation components reads:

Functional Model	**Observation Types and Stochastic Models**	
$h + v = H + h \cdot \Delta m + \mathrm{NFEM}(\mathbf{p} \mid x, y),$ $\quad \text{with } \mathrm{NFEM}(\mathbf{p} \mid x, y) =: \mathbf{f}(x, y) \cdot \mathbf{p}$	Uncorrelated ellipsoidal height h observations. Covariance matrix $\mathbf{C}_h = \mathrm{diag}(\sigma^2_{h_i})$.	(2a)
$N_G(B, L)^j + v = \mathbf{f}(x, y)^T \cdot \mathbf{p} + \partial N_G(\mathbf{d}^j)$	Correlated geoid height observations. With a given real covariance matrix $\mathbf{C}_{N_G}$ or a $\mathbf{C}_{N_G}$ evaluated from a synthetic covariance function.	(2b)
$\xi^j + v = -\mathbf{f}_B{}^T / M(B) \cdot \mathbf{p} + \partial\xi(\mathbf{d}^j{}_{\xi,\eta})$	Observations of deflections from the vertical (η,ξ). Pairwise correlated or uncorrelated in case of astronomical observations. Correlated if derived from a gravity potential model.	(2c)
$\eta^j + v = -\mathbf{f}_L{}^T / (N(B) \cdot \cos(B)) \cdot \mathbf{p} + \partial\eta(\mathbf{d}^j{}_{\xi,\eta})$		(2d)
$H + v = H$	Uncorrelated standard height H observations with covariance matrix $\mathbf{C}_H = \mathrm{diag}(\sigma^2_{H_i})$.	(2e)
$C + v = C(\mathbf{p})$	Continuity condition equations (1d) introduced as uncorrelated so-called pseudo observations with accordingly small variances and high weights.	(2f)

With $\mathbf{f}_B$ and $\mathbf{f}_L$ we introduce the partial derivatives of **f**(x(B,L),y(B,L)) (2c) with respect to the geographical coordinates B and L. M(B) and N(B) mean the radius of meridian and normal curvature at a latitude B. The continuity of the resulting HRS representation NFEM(**p**|x,y)=**f**(x,y)^T·**p** over the meshes (fig. 3, thin blue lines) is automatically provided by the continuity equations C(**p**) (2f).

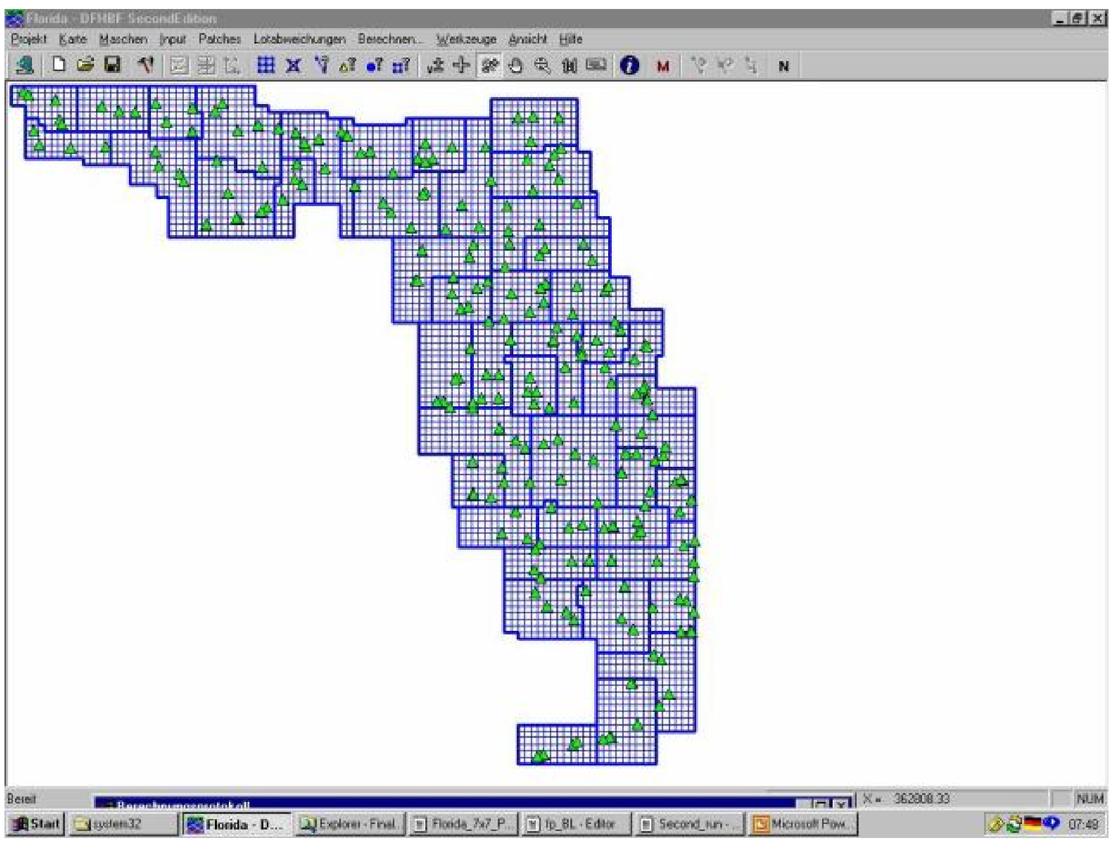


Fig. 3:
DFHRS-software at the example of the DFHRS_DB computation for Florida, USA
FEM-Meshes (thin blue lines), patches (thick blue lines) and fitting points (green triangles)

A number of identical fitting-points (B,L,h; H) are introduced by the observation equations (2a) and (2e) (fig. 3, green triangles). In the practice of DFHRS_DB evaluation, one or a number of different geoid-/GPM such as the EGM96/99 or EGG97 are used in a least squares estimation related to the mathematical model (2a-f), which is implemented in the DFHRS-software version 4.0 (fig. 3). To reduce the effect of medium- or long-wave systematic shape deflections, namely the natural and stochastic "weak-shapes", in the observations N and (ξ,η) from geoid- or GPM models, these observations are subdivided into a number of patches (fig. 3, thick blue lines). These patches are related to a set of individual parameters, which are introduced by the datum parametrizations $\partial N_G(\mathbf{d}^j)$ (2b) and $(\partial\xi(\mathbf{d}_{\xi,\eta});\partial\eta(\mathbf{d}^j_{\xi,\eta}))$ (2c, d). In this way, it is of course possible to introduce geoid height observations and vertical deflections from any number of different geoid- or GPM models in the same area, or observed vertical deflections.

Fig. 2 above gives an overview about the DFHRS_DB computed all over Europe in different accuracy classes concerning the respective DFHRS-correction (1). For the five German states, shown in the hatched yellow area of fig. 2, 1_cm DFHRS_DB, and in addition a continuous (1-3) cm DFHRS_DB all over Germany were computed. In that context two different kind of powerful block-inversions algorithms were developed and implemented into the DFHRS software, in order to avoid a limit for the number of unknowns in the normal-equation-matrix, which is else set by main storage of the PC. Other (1-3) cm DFHRS_DB were computed within the DFHRS project for Luxembourg, Estonia, Latvia, Lithuania, West Spain and Hungary, frequently in diploma and master thesis at HSKA and at different external institutions (fig. 2).

In 2004 the DFHRS-concept was applied for the evaluation of a 1_dm DFHRS_DB for Albania (geoid), and a closed and continuous 1_dm DFHRS for Europe in total (quasi-geoid), which is presently the most precise HRS for Europe (fig. 2). Outside Europe DFHRS_DB were computed for Namibia, Africa, for Tanzania, Africa and for Florida, USA (fig. 3).

The DFHRS_DB can also be used to setup the height transformation message declared in the new RTCM 3.0 correction data standard.

3. Physical Observation Components and Parametrisation - 2nd and present stage of the DFHRS concept further developments

The extension of the DFHRS-concept and -software to physical observation types - such as e.g. terrestrial, air- or space-borne gravity measurements (terrestrial gravity meter, see fig. 4), or physical observation types taken from geopotential models (GPM) - is based on a regional spherical cap harmonic parametrization (SCH) [6] of the earth's gravitational potential V. The benefit of SCH with a local cap pole and a limited cap size area, instead of an ordinary global spherical harmonic on (OSH), is that the same resolution of V can be achieved with SCH by a much less number of parameters than by OSH.

Fig. 4a:
Gravity meter for a terrestrial or airborne observation of the gravity vector
$\mathbf{g}^{\mathrm{LAV}} = [0,0,-g_P]^T$

Fig. 4b:
Zenith camera. Here the Model Startracker CT-602, Ball Aerospace used in Aircraft Navigation

E.g. for a 2 mm resolution for the HRS, a degree of 7200 for the OSH parametrization by ($C_{n,m}$; $S_{n,m}$) is required, while for a cap size area of 100 km a degree of about $k_{max} = 80$ should be enough for HRS in case for a SCH parametric model. But for the exploitation of the gravity observations accuracy of 0.02 mgal $k_{max} = 150 - 200$ in (4c) is necessary, which in analogy to OSH also higher than that for the HRS itself. So SCH in a degree, which sis adequate for observed gravity values, is the key for enabling the computation of high resolution HRS in the 2nd stage of the DFHRS research and development, meaning an integrated over-determined HRS-computation with geometrical and physical observations. The representation of the gravitational potential V of the earth in terms of SCH with parameters $(C'_{n(k),m}, S'_{n(k),m})$ reads ([6], [8]):

$$V(r,\lambda',\theta') = \sum_{k=0}^{k\,max} \left(\frac{a}{r}\right)^{n(k)+1} \sum_{m=0}^{k} (C'_{n(k),m} \cdot \cos m\lambda' + S'_{n(k),m'} \cdot \sin m\lambda') \cdot P'_{n(k),m}(\cos\theta') \quad . \tag{3}$$

Here the space position refers to the triple of spherical cap coordinates (r, λ', θ').

In the frame of that article, the observation equation for terrestrial and air- or space-borne gravity observations g_P is briefly worked out. For more details, and as concerns further physical observation types, related to (3), it is referred to [6], [7] and [8]. The gravity observation g_P at the earth surface, taken with a gravity meter (fig. 4), is referring to the local astronomical vertical system (LAV), and so we have for the respective observed three-dimensional gravity vector in total:

$$\mathbf{g}^{LAV} = [0,0,-g_P]^T \qquad \text{- Original gravity observation and vector.} \tag{4a}$$

The astronomical vertical ($\Phi = B + \xi$, $\Lambda = L + \eta / \cos(B)$) is set up by the ellipsoidal vertical (B,L) and the deflections from the vertical (ξ, η). The original vector $\mathbf{g}^{LAV}$ (4a) is first rotated to the earth-centred earth-fixed system (ECEF) using (Φ, Λ). In that coordinate frame, the centrifugal parts are removed, and so the original observation (4a) is strictly reduced with respect to the vertical deflections ("topography") and to the centrifugal acceleration. After a further rotation to the local geodetic vertical system (LGV) related to the cap sphere, we arrive at

$$\mathbf{g}_{red}^{LGV} = [g_N, g_E, g_r]^T \qquad \text{- Reduced gravity observation vector.} \tag{4b}$$

The observation vector $\mathbf{g}_{red}^{LGV}$ (4b) is further rotated as $\mathbf{g}_{grav}^{SCH}$ to the SCH-representation frame (3) with

$$\mathbf{g}_{grav}^{SCH} = [\frac{1}{r} \cdot \frac{\partial V}{\partial \theta'}, \frac{1}{r \cdot \sin\theta'} \cdot \frac{\partial V}{\partial \lambda}, \frac{\partial V}{\partial r}]^T \qquad \text{- Hypothesis free parametrization of the final } \mathbf{g}_{grav}^{SCH} \tag{4c}$$

The "vertical" and principal component of the finally reduced observed gravity observation $\mathbf{g}_{grav}^{SCH}$ is related to the third parametric component in (4c), and so we have the following observation equation for a gravity observation in the integrated DFHRS approach:

$$g_{grav\,r}^{SCH} = \sum_{k=0}^{\infty} \left(\frac{a}{r}\right)^{n(k)+1} \frac{(n(k)+1)}{r} \sum_{m=0}^{k} (C'_{n(k)),m} \cdot \cos m\lambda' + S'_{n(k),m} \cdot \sin m\lambda') \cdot P_{n(k),m}(\cos\theta') + dg(\mathbf{d}_g) \tag{4d}$$

Finally the condition equations (5) are introduced as pseudo observations over the mesh-grid (fig. 3) with high weights.

Functional Model

$$0 + v_{\Delta N} = N(C'_{n(k),m}, S'_{n(k),m}) - (\mathbf{f}^T \cdot \mathbf{p} + \Delta m \cdot h) \quad (5)$$

Observation Types and Stochastic Model

Condition equations "HRS from N(SCH) = NFEM(**p**)" as uncorrelated pseudo observations with small variances and high weights.

Equations (5) relate the HRS, as represented by $(C'_{n(k)m}, S'_{n(k),m})$, back to the classical and standardized DFHRS_DB parameters **p** and Δm and enable to set up the DFHRS standard correction (1). So the polynomial NFEM-parameters **p** may remain the main unknowns, and the standardized DFHRS_DB content can be kept for a HRS surface representation.

4. Outlook

The new integrated DFHRS approach ((2a-f), (3), (4a-d),5) is further developed and tested out in two projects at Karlsruhe University of Applied Sciences (HSKA), namely Saarland and Baden-Württemberg.

The representation of the terrestrial gravity measurements, fig. (4a) in the adequate resolution of the accuracy level of 0.01-0.02 mgal is achieved by a degree k of the SCH model (3), which is depending on the SCH area size. So the information content of original gravity observations is implicitly exploitable on the same level as e.g. in the Stokes family of approaches. For extended areas and a large number of unknowns a block-matrix based inversion of the normal equation matrix, as developed for the computation of DFHRS_DB Europe (fig. 2), is necessary to avoid a restriction to the number of unknowns due to the PC RAM.

The integrated DFHRS adjustment approach ((2a-f), (3), (4e), (5)) is presently improved with respect to the numerical solution of the weak conditioned normal equations in case of higher SCH degrees k. Additionally the investigations with the above mentioned real data sets of Baden-Württemberg and Saarland are dealing with the additional parameter modelling $dg(\mathbf{d}_g)$ (4e), and with the question concerning the area-size and the density of the original terrestrial gravity observations (4a), which are – depending on the design of other observation groups (2a-f) and the GPM information – required to achieve e.g. a 1_cm DFHRS in any area. These topics are part of the present research work. In 2007 the completing of the DFHRS software version 5.0 with respect to the integrated approach in planned.

Due to its characteristics as adjustment concept, the integrated DFHRS-approach ((2a-f), (3), (4e), (5)) can be used to analyse and optimize the observation and network design (1st, 2nd and 3rd order design) with regard to accuracy related target functions based to the HRS-parametrization DFHRS(**p**,Δm|B,L,h) and its covariance matrix. In that context presently the question of the optimal mixed design of identical points, terrestrial gravity observations (fig. 4a) and vertical deflections observations from modern automated zenith-cameras (fig. 4b) is of great interest in HRS-computations. That topic will be another research area starting in 2007.

A third topic of research will be dealing with some alternative numerically stable parametrizations to SCH for physical observations.

The above further developments are just some examples for present and near future research and development work in the DFHRS project at HSKA.

5. References

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