DFHRS-based Ulaanbaatar Region Quasi-Geoid Baltics Height System 2017 (1st Version)

Report

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Karlsruhe, 10-July-2017

Content

1. Introduction

The DFHRS (Digital-Finite-Element-Height-Reference-Surface) research and development project, German word DFHBF, aims at the computation of height reference surfaces (HRS) [www.dfhbf.de].

A HRS is represented by the height N of the HRS above the reference ellipsoid (fig. 1).

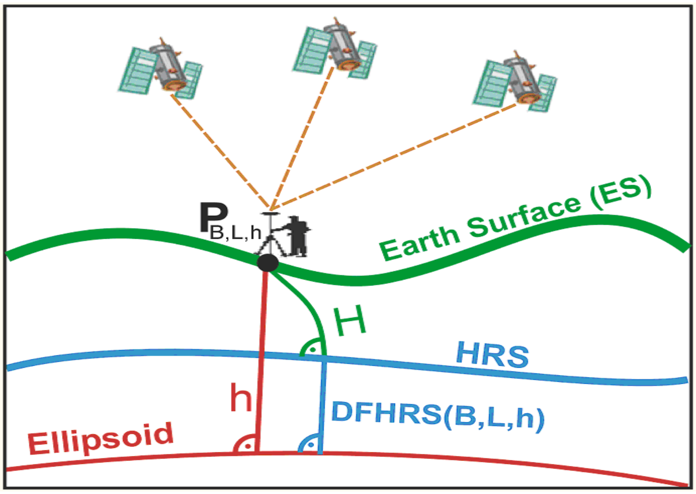

Figure 1. Principle of GNSS-based height determination: H = h – DFHRS(B,L,h)

The main practical target of the DFHRS project is to enable for Global Navigation Satellite Systems (GNSS), such as GPS, GLONASS, BEIDOU, GALILEO and respective PPP-K or DGNSS-positioning services (SAPOS, www.sapos.de in Germany) based height determination, the direct conversion of the ellipsoidal GNSS height h determined at the earth surface (ES), into the physical earth gravity field based standard “sea-level” height H = h – N (fig. 1).

Depending on the height system type, the physical heights H are called orthometric, normal or spheroidal normal heights (NN-heights), and the respective HRS is called geoid, quasi-geoid or NN-surface. In the DFHRS concept version 4.x, a continuous polynomial surface over of a grid of finite element meshes (FEM) with polynomial parameters **p** (fig. 3, thin blue lines) is used as a carrier function N=N(B,L,h) for the HRS. The FEM surface of the HRS is therefore called NFEM(**p**|B,L). The above HRS-types show weak or missing dependences of the HRS height N from h, which is treated in the mathematical computation model of the DFHRS approach below, and therefore already included in the final HRS representation NFEM(**p**|B,L). For some old height systems H a scale-difference factor Δm has to be considered in addition, so that the DFHRS-correction DFHRS (fig. 1) consists of two parts. The principle of a GNSS-based height determination H (fig. 1), requires to submit the GNSS-height h to the DFHRS(B,L,h)-correction, and it reads:

$$H = h - DFHRS(p, \Delta m \,|\, B, L, h) = h - (NFEM(p \,|\, B, L) + \Delta m \cdot h) \quad . \qquad (1)$$

The DFHRS-correction DFHRS(B,L,h) is provided by means of a DFHRS database (DFHRS_DB), which contains the HRS parameters (**p,** Δm) together with the mesh-design (fig. 3) information. DFHRS_DB have become an industrial and user standard for all GNSS-receiver types worldwide and a new kind of modern geo-data product in the GNSS navigation age.

In the 1st stage of the DFHRS approach development, geoid- or geopotential model (GPM) heights N, observed astronomical or geoid/GPM-model based deflections of the vertical (ξ,η) in any number of groups, and fitting points (B,L,h; H) were exclusively used as observation groups in a common least squares computation for the evaluation of the DFHRS_DB parameters **p** and Δm. The mathematical model for these observations is given by formulas (2a-f). In case of an adequate stochastical model, the use of gravity-based geoid-/GPM model information is equivalent to the use of the original observed gravity values g. The mathematical model for the computation of the DFHRS_DB parameters (**p,** Δm) using the above so-called geometrical part of observation components reads:

Functional Model	**Observation Types and Stochastic Models**	
$h + v = H + h \cdot \Delta m + NFEM(\mathbf{p} \mid x, y)$, with $NFEM(\mathbf{p} \mid x, y) =: \mathbf{f}(x, y) \cdot \mathbf{p}$	Uncorrelated ellipsoidal height h observations. Covariance matrix $\mathbf{C}_h = diag(\sigma^2_{h_i})$.	(2a)
$N_G(B,L)^j + v = \mathbf{f}(x,y)^T \cdot \mathbf{p} + \partial N_G(\mathbf{d}^j)$	Correlated geoid height observations. With a given real covariance matrix $\mathbf{C}_{N_G}$ or a $\mathbf{C}_{N_G}$ evaluated from a synthetic covariance function.	(2b)
$\xi^j + v = -\mathbf{f}_B{}^T / (M(B) + h) \cdot \mathbf{p} + \partial\xi(\mathbf{d}^j{}_{\xi,\eta})$	Observations of deflections from the vertical (η,ξ). Pairwise correlated or uncorrelated in case of astronomical observations. Correlated if derived from a gravity potential model.	(2c)
$\eta^j + v = -\mathbf{f}_L{}^T / (N(B) + h) \cdot \cos(B)) \cdot \mathbf{p} + \partial\eta(\mathbf{d}^j{}_{\xi,\eta})$		(2d)
$H + v = H$	Uncorrelated standard height H observations with covariance matrix $\mathbf{C}_H = diag(\sigma^2_{H_i})$.	(2e)
$C + v = C(\mathbf{p})$	Continuity condition equations (1d) introduced as uncorrelated so-called pseudo observations with accordingly small variances and high weights.	(2f)

With $\mathbf{f}_B$ and $\mathbf{f}_L$ we introduce the partial derivatives of **f**(x(B,L),y(B,L)) (2c) with respect to the geographical coordinates B and L. M(B) and N(B) mean the radius of meridian and normal curvature at a latitude B.

The continuity of the resulting HRS representation $NFEM(\mathbf{p}|x,y)=\mathbf{f}(x,y)^T \cdot \mathbf{p}$ over the meshes (fig. 3, thin blue lines) is automatically provided by the continuity equations C(**p**) (2f).

A number of identical fitting-points (B,L,h; H) are introduced by the observation equations (2a) and (2e) (fig. 3, green triangles). In the practice of DFHRS_DB evaluation, one or a number of different geoid-/GPM such as the EGM96/99, EGG97 and at present EGM 2008 are used in a least squares estimation related to the mathematical model (2a-f), which is implemented in the DFHRS-software 4.4 (fig. 3). To reduce the effect of medium- or long-wave systematic shape deflections, namely the natural and stochastic “weak-shapes”, in the observations N and (ξ,η) from geoid- or GPM models, these observations are subdivided into a number of patches (fig. 3, thick blue lines). These patches are related to a set of individual parameters, which are introduced by the datum parametrizations $\partial N_G(\mathbf{d}^j)$ (2b) and $(\partial\xi(\mathbf{d}_{\xi,\eta}); \partial\eta(\mathbf{d}^j{}_{\xi,\eta})$ (2c,d). In this way, it is of course possible to introduce geoid height observations and vertical deflections from any number of different geoid- or GPM models in the same area, or observed vertical deflections.

Fig. 2 gives an overview about the DFHRS_DB computed all over Europe in different accuracy classes concerning the respective DFHRS-correction (1). For the five German states, shown in the hatched yellow area of fig. 2, 1_cm DFHRS_DB, and in addition a continuous (1-3) cm DFHRS_DB all over Germany were computed. Other (1-3) cm DFHRS_DB were computed within the DFHRS project for Luxembourg, Estonia, Latvia, Lithuania, West Spain and Hungary, frequently in diploma and master thesis at HSKA and at different external institutions (fig. 2). In 2012 the DFHRS-concept was applied for the evaluation of a 1_dm DFHRS_DB for Albania (geoid), and a closed and continuous 1_dm DFHRS for Europe in total (quasi-geoid), which is presently the most precise HRS for Europe (fig. 2).

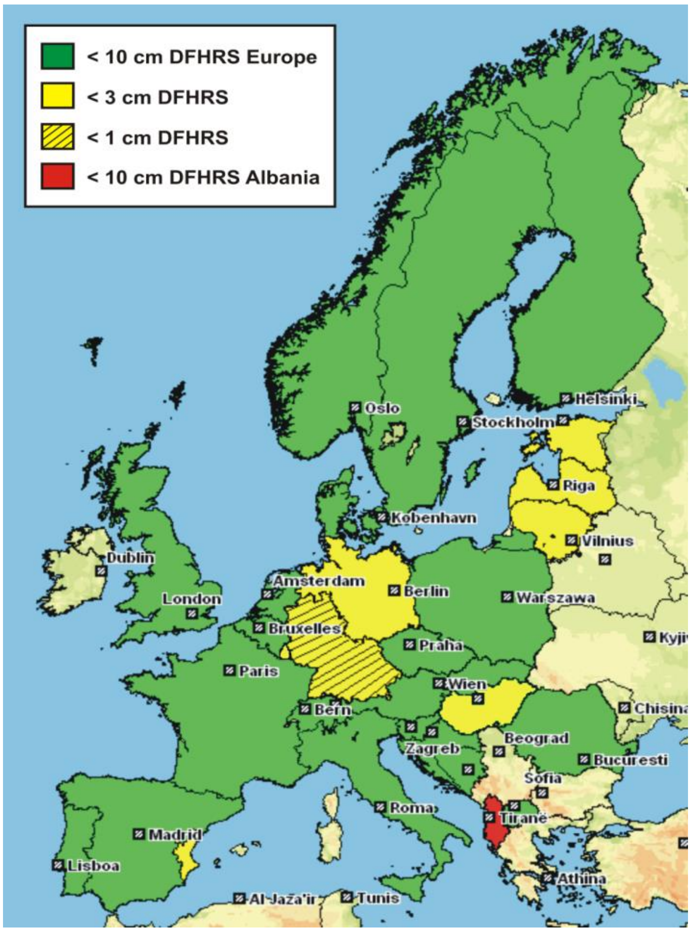

Figure 2. Overview on DFHRS_DB computed all over Europe

Outside Europe DFHRS_DB were computed for two African states and for Florida, USA (fig. 3). Further DFHRS_DB outside Europe were computed for Namibia, Brazil and Moldova (www.moldpos.eu ; www.dfhbf.de) .

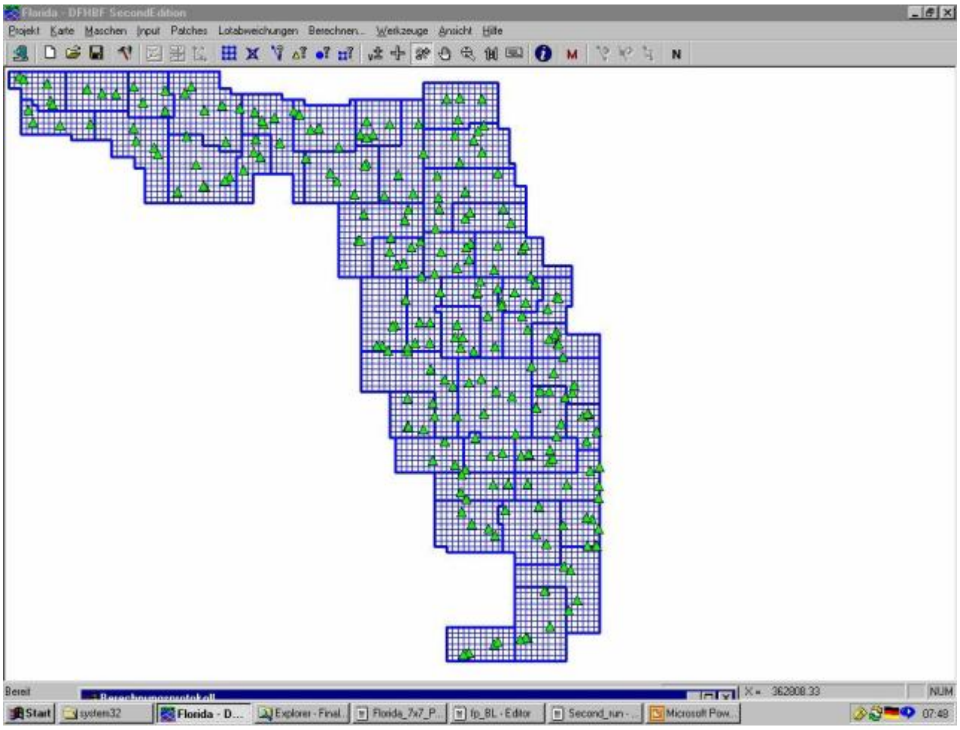

Figure 3. DFHRS-software at the example of the DFHRS_DB computation for Florida, USA
FEM-Meshes (thin blue lines), patches (thick blue lines) and fitting points (green triangles)

2. DFHRS-based Ulaanbaatar Region Quasi-Geoid for the Baltics Height System 2017 (1^{st} Version)

2.1 Used data

In order to compute the DFHRS (digital finite element height reference surface) database for Ulaanbaatar (see the output protocol in the appendix) the following data were used:

51 Identical points (ellipsoidal heights h and normal heights H in Baltic Height system) together with the EGM2008 geopotential model.

EGM2008 is a spherical harmonic model of the earth's external gravitational potential in degree and order of 2160, with additional spherical harmonic coefficients extending up to degree of 2190 and order of 2160 that offers a spatial resolution of 9 km. EGM2008 incorporates improved 5x5 min gravity anomalies, altimetry-derived gravity anomalies and has benefited from the latest GRACE based satellite solutions [3].

2.2 Meshing of the Area

For meshing the area, mesh size of 5x5 km was chosen (fig. 4, thin blue lines). Total amount of meshes – 1536. To reduce the effect of medium or long-wave length, systematic shape deflections, namely the natural and stochastic “weak shapes”, in the observations N and (ξ,η) from geoid or GPM models, the mathematical model of the DFHRS concept allows subdividing any geoid model that is used into a number of so called “geoid-patches”; see (fig. 4, thick blue lines). The total number of patches is 5.

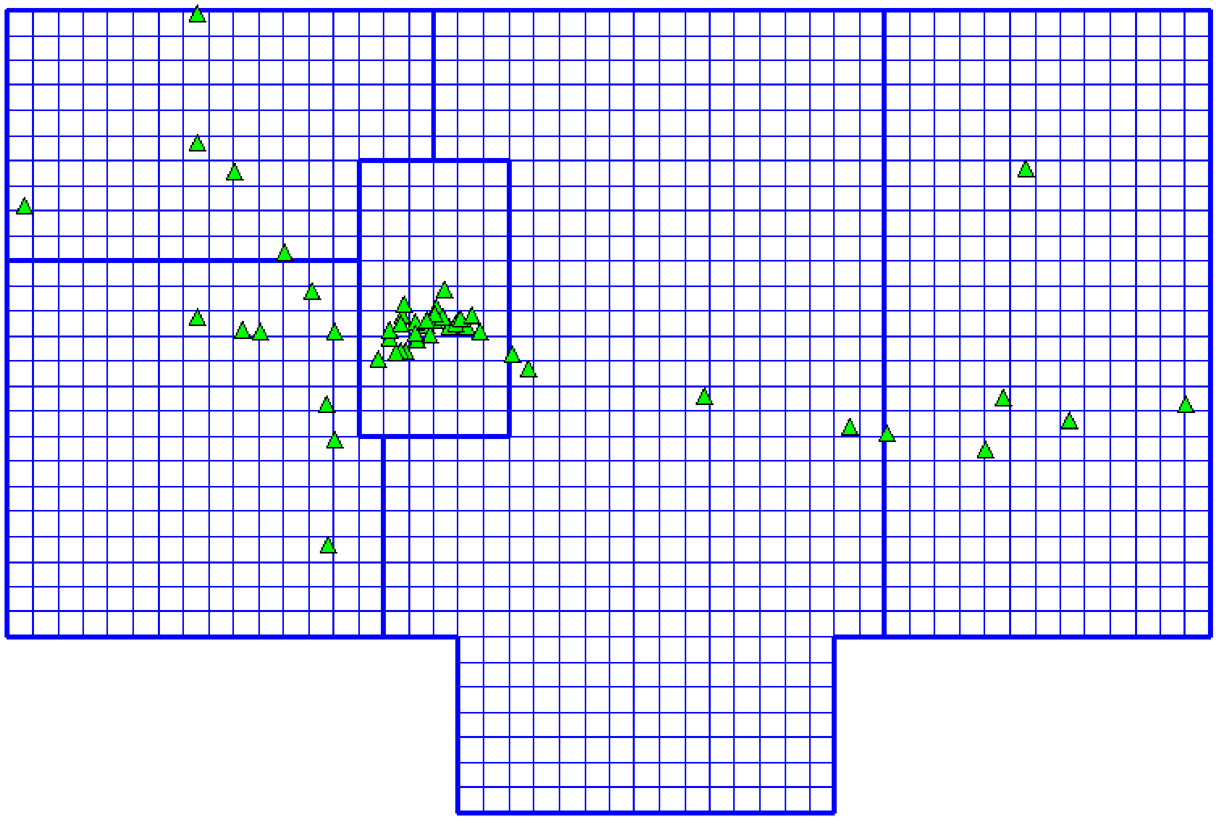

Figure 4. Computation design of DFHRS (meshes – thin blue lines, patches – thick blue lines, fitting points – green triangles)

2.3 Results

2.3.1 Adjustment and Datum-Parameter Settings

As already mentioned (see fig. 4), the area for the elimination of systematics was subdivided into 5 patches with their own data parameters. One patch must contain at least 4 fitting points. As points of the region are not homogenously located, patches were not introduced in approximately the same size, but according to points location. As geoid datum 3 translations and 3 rotations were introduced, additionally deflections of the vertical from the EGM2008 model were used (see fig. 5).

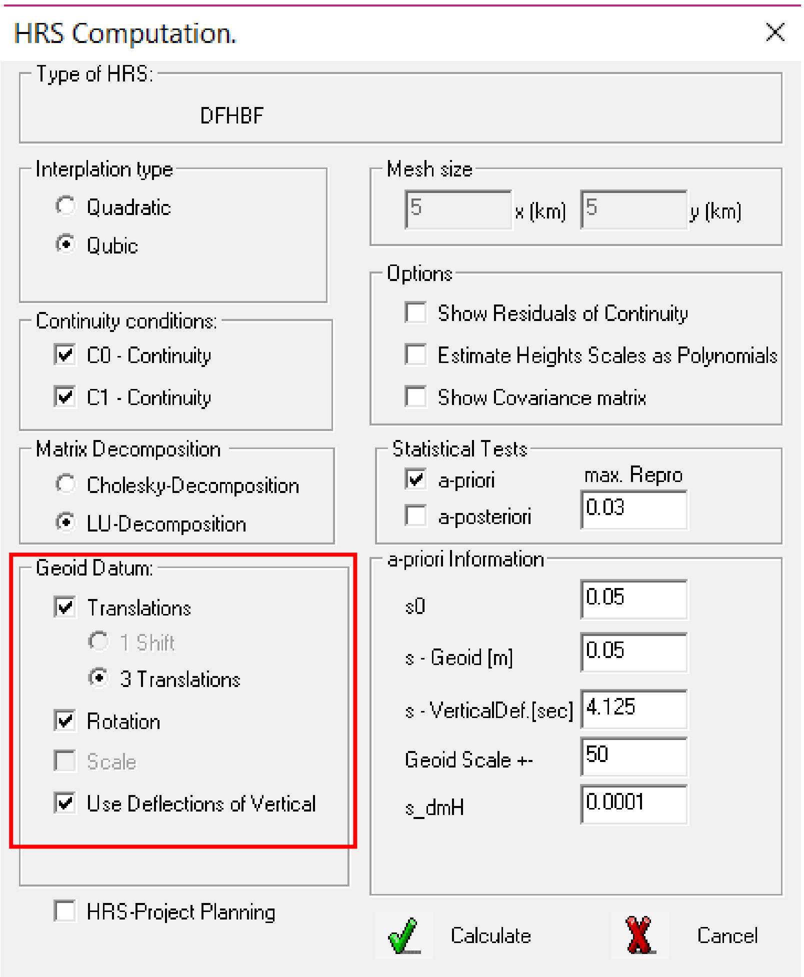

Figure 5. DFHRS-software 4.4 computation dialog

2.3.2 Adjustment results

The identical points and the EGM2008 geoid undulations were introduced together with the continuity conditions into a least squares estimation of the so-called "DFHRS production". The calculation was done using the DFHRS v. 4.4. Software

49 normal height points H of the Baltic heights system could be used and were confirmed in the statistical testing (data-snooping) with the assumed standard deviation of 1 cm. Only the five points 22, 230, 1757, 505 and 70 had to be introduced and were confirmed with the higher standard deviation of 3 cm. 2 points – HUMU and 1710 were excluded from the computations because of gross errors of 20.0 and -20.8 cm respectively. The residuals are depicted on fig. 6.

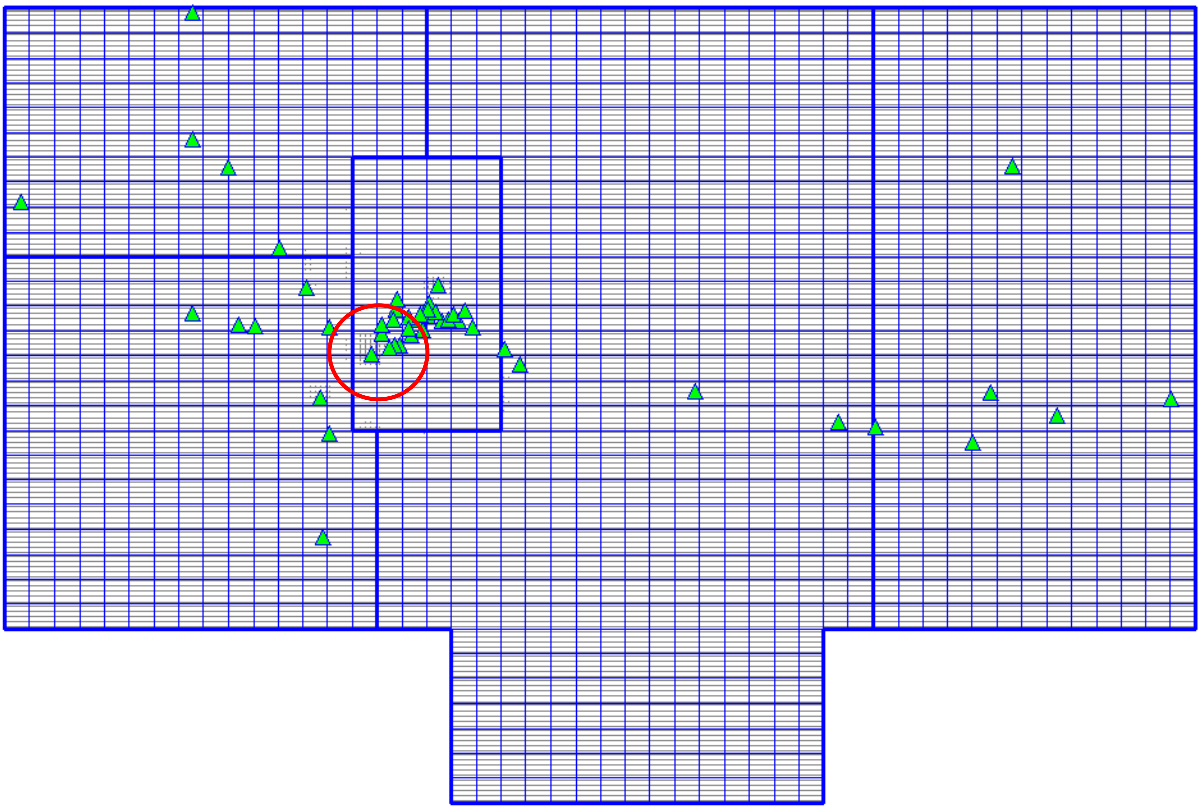

Figure 6. The residuals of the points (the highest residuals are depicted with thin vertical lines see. red circle)

The adjustment protocol with the residuals, statistics testing of the height points are depicted in table 1.

Table 1: Final DFHRS software adjustment protocol

```
################################################################################
#                             DFHBF SecondEdition                              #
#                             Calculation protocol                             #
################################################################################

Calculation details:  09.07.2017, 04:20 Uhr.
Computation time:        30 min. 20 sek.

Project name:           UB_BHS
Project directory:      C:\DFHRS\UB_BHS
Type of calculated HBF: General DFHBF
Interpolation type:    cubic
Geoid datum translation:    3 Rotations and 3 Translation

################################################################################
#                                                                              #
#                             Observations                                     #
#                                                                              #
################################################################################

>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<
>>>                             <<<
>>>   (Identical) Fitting points <<<
>>>                             <<<
>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<
```

```
--------------------------------------------------------------------------------
Pointnumber   Height/Targetsys.    Accuracy.  Height/Sourcesys.    Accuracy.  Used?
                           [m]           [m]                  [m]          [m]
--------------------------------------------------------------------------------
Total number of heights in the target system:                     49
Total number of unused heights in the target system:              49
Total number of unused heights in the source system:              49
--------------------------------------------------------------------------------

>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<
>>>                            <<<
>>>  Geoidundulations: EGM2008 <<<
>>>                            <<<
>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<

--------------------------------------------------------------------------------
Specified Accuracy  :      0.050 m
Accuracy geoids deflections of vertical[sec] :    4.125
--------------------------------------------------------------------------------
Total number geoid undulation:                                          32625
Total number of used geoid undulations:                                 32625
Total number of used EGG deflections of the vertical:               65250
--------------------------------------------------------------------------------
################################################################################
#                                                                              #
#                                 Adjustment                                   #
#                                                                              #
################################################################################

Observations:
--------------------------------------------------------------------------------
Number of unused heights in the target system:             49
Number of unused heights in the source system:            49
Number of used height differences:           0
Number of used geoid undulations:          32625
Number of used EGM2008-Deflections of the vertical:        65250
Number of continuity conditions:              18004
--------------------------------------------------------------------------------
Unknowns:
--------------------------------------------------------------------------------
Number of heights in the target system:                   49
1 Scale difference:                             1
Number of polynomial parameters:              13050
--------------------------------------------------------------------------------
Redundancy (Observations-Unknown):            102848
--------------------------------------------------------------------------------

>>>>>>>>>>>>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<<<<<<
>>>                                            <<<
>>>  Test the identical fixed points           <<<
>>>                                            <<<
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<<<<<

--------------------------------------------------------------------------------
 Characteristics:
   EV:     Redudancy factor
   NV:     Normalized residuals, test size a priori
   t_post: Test size a posteriori
   GF:     Estimated gross error is issued in case of exceeding
           the critical value by NV, bzw. t_post.
--------------------------------------------------------------------------------
Probability of error Alpha: 5 %
Critical value a priori:       3.841552  degrees of freedom:  infinity
Critical value a posteriori:    3.841549  degrees of freedom:     102848
--------------------------------------------------------------------------------
Point number  Height/Target sys.     Res.       EV        NV    t_post   REPRO
                        [m]          [m]         [%]
--------------------------------------------------------------------------------
          ULBA       1266.703      0.01886     36.33      3.1     79.2  -0.052
          1747       1313.276      0.00359     35.46      0.6     14.8  -0.010
```

1731	1608.961	0.00154	25.10	0.3	7.5	-0.006
1598	1418.122	0.00730	21.45	1.6	39.0	-0.034
1573	1495.333	0.00075	12.48	0.2	5.2	-0.006
24	1414.530	-0.00301	35.53	0.5	12.4	0.008
10	1336.377	-0.00727	27.59	1.4	34.2	0.026
22	1228.700	-0.03672	40.67	1.9	47.7	0.090
19	1428.082	0.00508	20.56	1.1	27.6	-0.025
9509	1021.863	0.00114	8.46	0.4	9.6	-0.013
Yarm	1310.379	-0.00122	41.17	0.2	4.7	0.003
Nogo	1354.608	0.00611	31.11	1.1	27.0	-0.020
Huld	1377.522	-0.00552	37.28	0.9	22.3	0.015
DoNi	1271.757	-0.01404	34.96	2.4	59.3	0.040
1757	1220.927	0.03211	46.47	1.6	38.8	-0.069
1717	1395.812	0.00466	33.71	0.8	19.8	-0.014
1682	1456.714	-0.00013	23.59	0.0	0.7	0.001
1674	1433.712	-0.00582	30.49	1.1	26.0	0.019
1670	1359.465	-0.00107	32.00	0.2	4.6	0.003
1412	1466.031	0.00583	25.89	1.1	28.2	-0.023
5051	1433.680	-0.00121	32.26	0.2	5.2	0.004
297	1365.216	-0.00312	28.37	0.6	14.4	0.011
270	1393.994	-0.00142	9.27	0.5	11.4	0.015
154	1261.955	0.00612	33.82	1.1	25.9	-0.018
2	1294.164	-0.00100	42.26	0.2	3.8	0.002
4	1283.585	-0.00093	40.31	0.1	3.6	0.002
92	1264.917	0.00580	40.19	0.9	22.5	-0.014
139	1312.021	-0.00427	41.85	0.7	16.2	0.010
164	1313.365	-0.00078	41.72	0.1	3.0	0.002
224	1333.410	0.00354	40.59	0.6	13.6	-0.009
227	1297.124	0.00156	42.91	0.2	5.8	-0.004
230	1264.222	0.07873	85.69	2.8	71.3	-0.092
235	1317.683	-0.02255	40.31	3.6	90.7	0.056
236	1280.370	0.00125	40.68	0.2	4.8	-0.003
246	1283.432	0.00016	43.46	0.0	0.6	-0.000
260	1284.586	-0.00704	42.12	1.1	26.7	0.017
355	1313.854	-0.00757	38.77	1.2	30.0	0.020
502	1360.459	-0.01995	37.17	3.3	83.0	0.054
504	1306.889	0.00416	43.11	0.6	15.6	-0.010
507	1318.669	-0.00313	43.23	0.5	11.7	0.007
510	1343.038	-0.00042	41.92	0.1	1.6	0.001
511	1305.459	0.00990	42.73	1.5	37.4	-0.023
512	1358.381	0.00958	39.97	1.5	37.5	-0.024
515	1293.477	0.00010	42.16	0.0	0.4	-0.000
518	1332.319	0.00249	42.98	0.4	9.3	-0.006
519	1288.146	0.00427	41.57	0.7	16.3	-0.010
528	1276.513	0.00035	42.51	0.1	1.3	-0.001
70	1439.719	0.04970	45.90	2.4	61.1	-0.108
505	1354.620	-0.05775	48.29	2.8	69.6	0.120

Excluded points from the computations are depicted in table 2.

Table 2. Eliminated Error Points

```
Probability of error Alpha: 5 %
Critical value a priori:        3.841552  degrees of freedom:  infinity
Critical value a posteriori:    3.841549   degrees of freedom:     102848
```

Point number	Height/Target sys. [m]	[m]	Res. [%]	EV	NV	t_post	REPRO
1710	1478.734		0.06712	32.44	11.8*	2.8	-0.207**
	!!! ---> GF:	-0.207 m <--- !!!					
HUMU	1457.023		-0.06497	32.52	11.4*	2.7	0.200**
	!!! ---> GF:	0.200 m <--- !!!					

3. Conclusions

The present DFHRS was calculated on the basis of the EGM2008 geoid and 49 identical reference points. The accuracy of the identical points was confirmed with 1.0 - 3.0 cm, so the QGeoid of the Ulaanbaatar region has an estimated 1-3 cm accuracy within the area of the outer ring polygon-line of the fitting-points.

The DFHRS_DB (QGeoid_Ulaanbaatar_2017.HBF) can be used by the software DFHBF-Tools to compute the QGeoid-height N, and so the Normal Heights H from the input of a 3D GNSS-position (B,L,h) or (x,y,z), and in order to set up a respective QGeoid 2017 grid for the Baltic Height System in the Ulaanbaatar Region.

Especially for the Northern and Southern region of the central patch of the network (fig. 4, 6) additional vertical observations are recommended, this also holds as well as in the North-West, South-West and North-East, South-East part of the area. In that way, the 1-3 cm accuracy (even a better one by these additional observations) will hold for the whole area (fig. 4,6).

Literature

[1] DFHBF-Website: www.dfhbf.de

[2] R. Jäger, J. Kaminskis, J. Strauhmanis, and G. Younis, "Determination of quasi-geoid as height component of the geodetic infrastructure for GNSS positioning services in the Baltic States," Latvian J. of Physics and Technical Sciences 3, 2012, pp. 5–15.

[3] Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2008). An Earth Gravitational model to degree 2160: EGM2008, General Assembly of the European Geosciences Union, Vienna, Austria

[4] International Center for Global Gravity Field Models http://icgem.gfz-potsdam.de/ICGEM/